# Transport transforms for signal analysis and machine learning

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**imagedatascience.com/transport**
**github.com/rohdelab/PyTransKit**

# "… we get by with a little help from our friends …"

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NIH R01 GM130825
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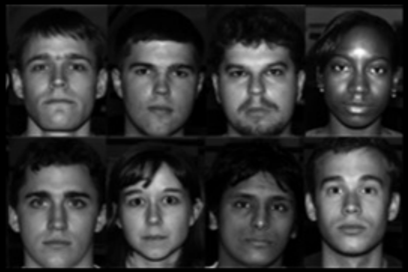
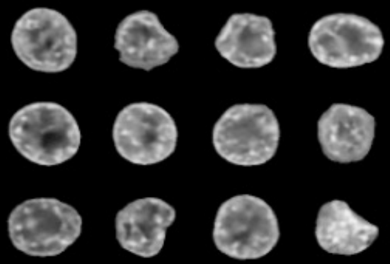
# Can we build smart systems cheaply with machine learning techniques?

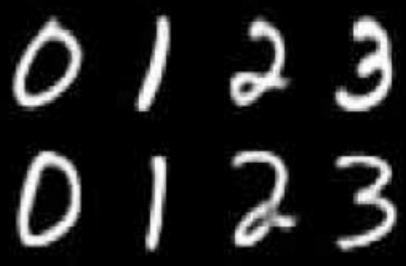
faces



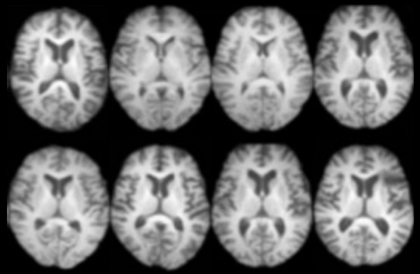
cells



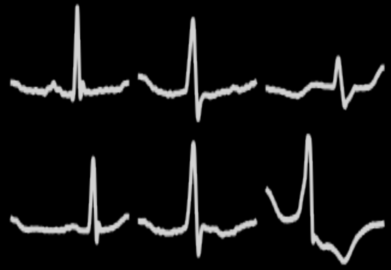
digits



brains



ECG



Example applications:

- Face recognition
- Medical diagnosis
  - (pathology, radiology)
- Optical character recognition
- EKG diagnosis
- Self driving cars
- Speech recognition
- …

What mathematical model shall we use?

How do we partition these data classes?

# Outline

- Modeling signal and image data
  - What are we talking about?
  - Transport model & problem statement
- Transport embeddings (representations, transforms)
  - 1D transform (CDT), ND (R-CDT, LOT),
  - Properties
- Applications
  - Signal/image classification
  - Transport-based morphometry

Matthew Thorpe
Caroline Mossmueller
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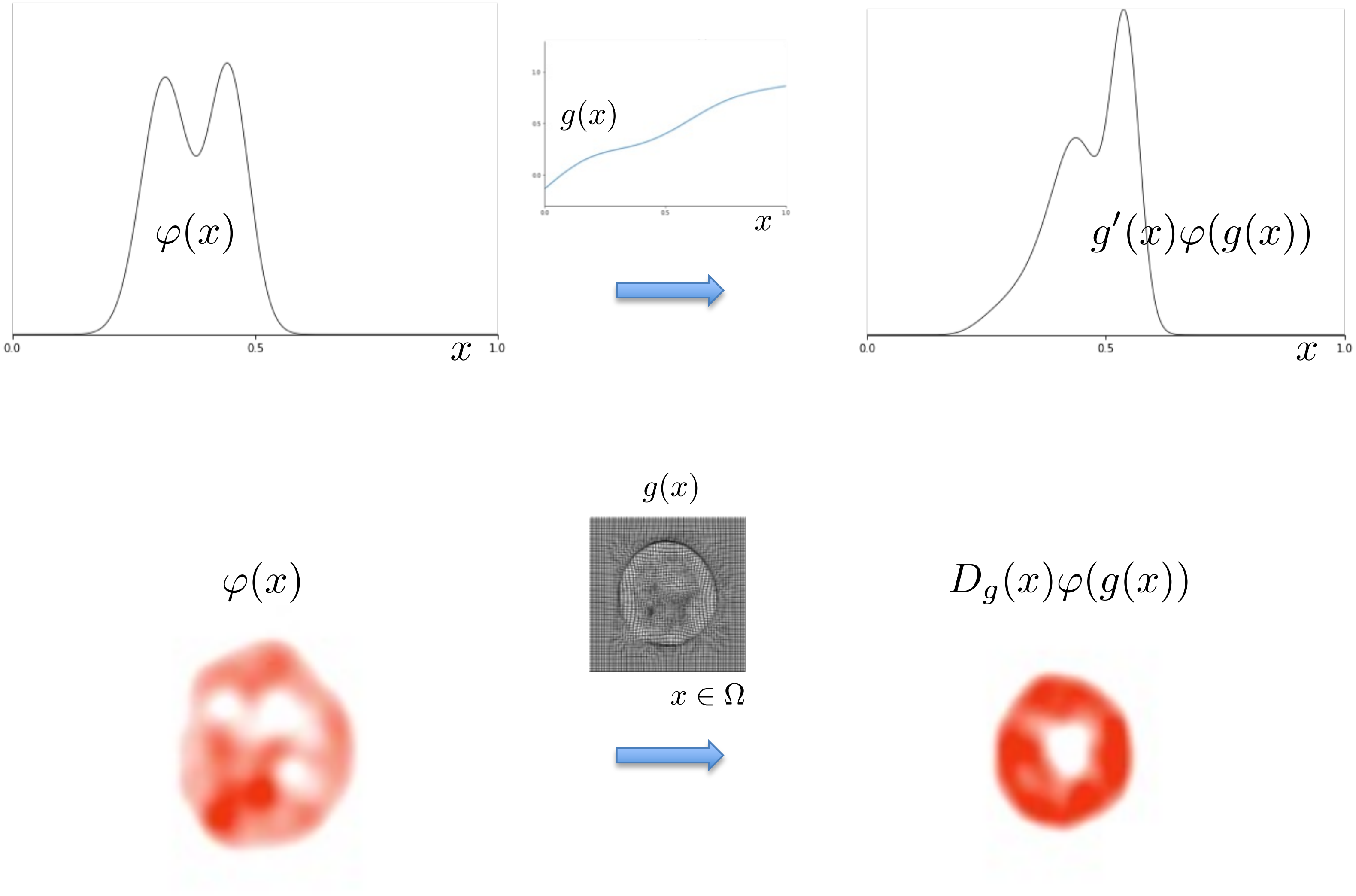
D. Slepcev
D. Nguyen
B. Schmitzer
...

# Modeling signal and image data

# Model for signal/image deformations (transportations)




Translation:

$$g(x) = x - \tau$$

Scaling:

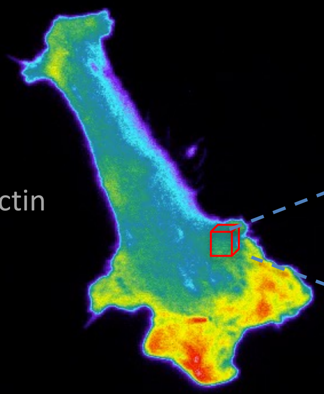
$$g(x) = ax$$

Nonrigid deformations:

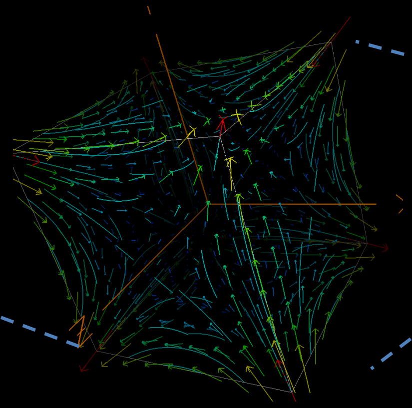
$$g(x) = \sum_k c_k \phi_k(x)$$

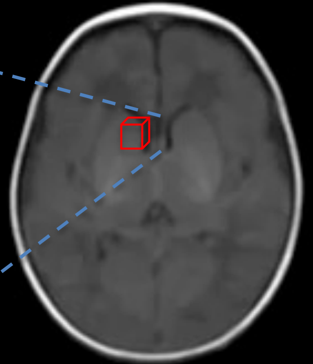
# What processes change signal/pixel intensity over time?

actin



Johnson et al, JCB, 2015





0 – 48 months

Nist lab, MNI

$\mathbf{v}$ flow/velocity

intensities not absolute

continuity equation

$$\underbrace{\frac{dI(x,y,z,t)}{dt}}_{\text{change in intensity at particular location}} = -\underbrace{\nabla\cdot\left(I(x,y,z,t)\mathbf{v}(x,y,z,t)\right)}_{\text{net flux tissue velocity}} + \underbrace{\cancel{(I, \ldots y, \ldots t)}}_{\text{source/sink term}}$$

# Transport with least action (optimal transport)

$I_0$ $\quad\underbrace{\qquad\qquad\qquad\qquad}_{I(x,t)}\quad$ $I_1$

$$\left.\begin{aligned} I(x,0) &= I_0(x) \\ I(x,1) &= I_1(x) \end{aligned}\right\}$$ sample images in a dataset

$$\frac{dI(x,t)}{dt} = -\nabla \cdot (I(x,y)v(x,t))$$

implies

$$I(x,t) = D_f(x) I_0(f(x,t))$$

$$f(x,t) = \int_0^t v(x,t)dt$$

**Theorem:** [Benamou, Brenier, Numer. Math, 2000]

$$W^2(I_0, I_1) = \inf_v \int_0^1 \int_\Omega I(x,t)|v(x,t)|^2 dx dt$$

$$f(x,t) = (1-t)x + t\nabla\phi$$

gradient of convex function

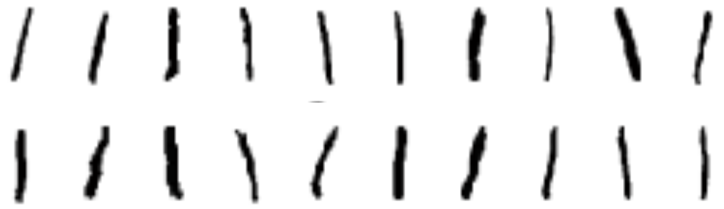
# Model for signal/image classes

An image class is modeled as

a set of transport maps $g_j \in \mathcal{G}$

applied to a template $\varphi^{(k)}$

$\mathbb{S}^1$ : class for digit 1

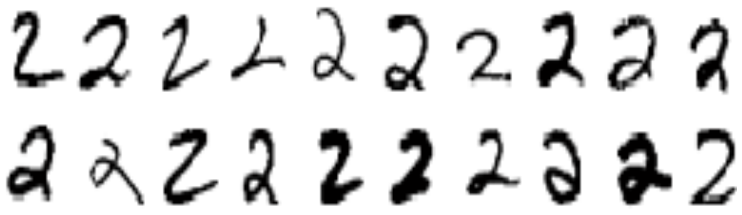


$$\mathbb{S}^{(1)} = \left\{ s_j^{(1)} | s_j^{(1)} = |\det J g_j| \varphi^{(1)} \circ g_j, \forall g_j \in \mathcal{G} \right\}$$

template for class 1 $\varphi^{(1)} =$ 

“confound”

$\mathbb{S}^2$ : class for digit 2

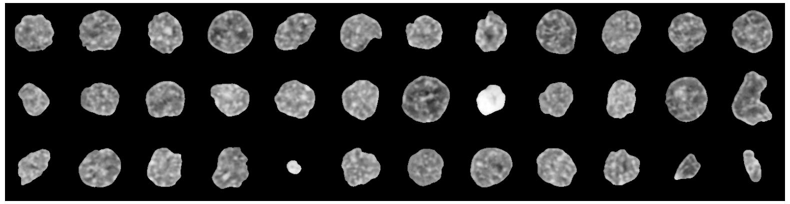


$$\mathbb{S}^{(2)} = \left\{ s_j^{(2)} | s_j^{(2)} = |\det J g_j| \varphi^{(2)} \circ g_j, \forall g_j \in \mathcal{G} \right\}$$

template for class 2 $\varphi^{(2)} =$ 

# A typical learning problem

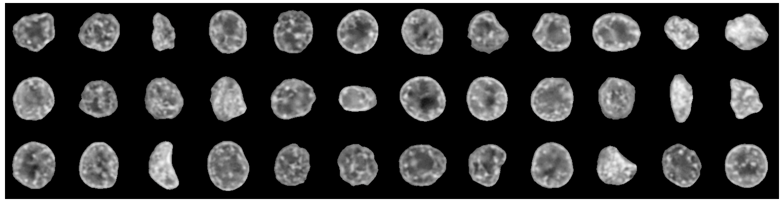
$\mathbb{S}^1$ : class 1 (benign cell nuclei)



$$\mathbb{S}^{(1)} = \left\{ s_j^{(1)} | s_j^{(1)} = |\det J g_j| \varphi^{(1)} \circ g_j, \forall g_j \in \mathcal{G} \right\}$$

$\varphi^{(1)} =$ unknown

$\mathbb{S}^2$ : class 2 (malignant)



$$\mathbb{S}^{(2)} = \left\{ s_j^{(2)} | s_j^{(2)} = |\det J g_j| \varphi^{(2)} \circ g_j, \forall g_j \in \mathcal{G} \right\}$$

$\varphi^{(2)} =$ unknown

$g_j \in \mathcal{G}$ unknown

# Model for 1D signal classes

$\mathbb{S}^{(1)}$: class for normal ECG heartbeats

$$\mathbb{S}^{(1)} = \{s_j^{(1)} | s_j^{(1)} = g'_j \varphi^{(1)} \circ g_j, \forall g_j \in \mathcal{G}\}$$

template for class 1: $\varphi^{(1)}$

$\mathbb{S}^{(2)}$: class for premature ventricular contraction beats

$$\mathbb{S}^{(2)} = \{s_j^{(2)} | s_j^{(2)} = g'_j \varphi^{(2)} \circ g_j, \forall g_j \in \mathcal{G}\}$$

template for class 2: $\varphi^{(2)}$

# Solution: Transport Transforms

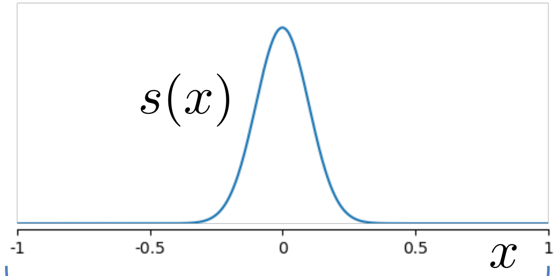
| signal space | | transform space | | signal space |
|---|---|---|---|---|
| problem hard to solve | ⇒ | problem easy to solve | ⇒ | answer in signal space |

estimation,
detection,
inverse problems,
classification

[Wang, Slepcev, Basu, Ozolek, Rohde, IJCV 2013]
[Kolouri, Tosun, Ozolek, Rohde, Pattern Recognition 2016]
[Kolouri, Park, Rohde, IEEE TIP, 2016]
[Park, Kolouri, Kundu, Rohde, ACHA 18]
[Rubaiyat et al, IEEE TSP, 20]
[Aldroubi, Li, Rohde, SaSiDa, 21]
[Shifat-E-Rabbi et al, Rohde, JMIV 21]
[Aldroubi et al, AIMS Foundations of Data Science, 22]

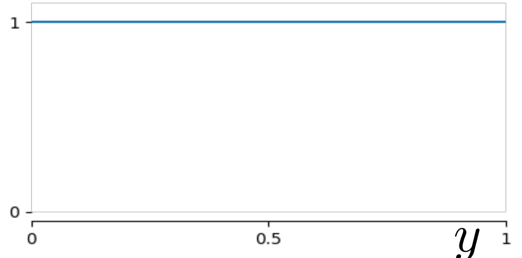
# Transport transforms

# CDT: invertible (nonlinear) signal transform

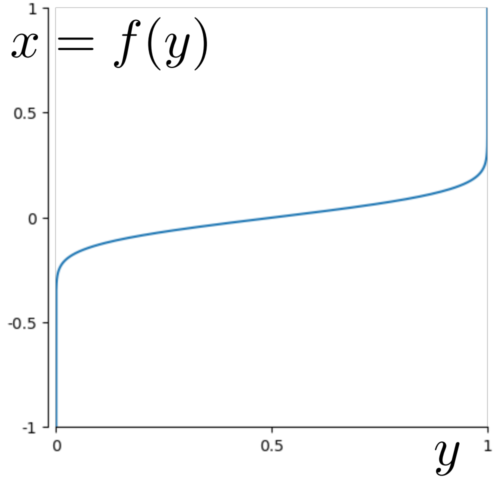



$x \in \Omega$

$s_0(y)$




$y \in \Omega_0$



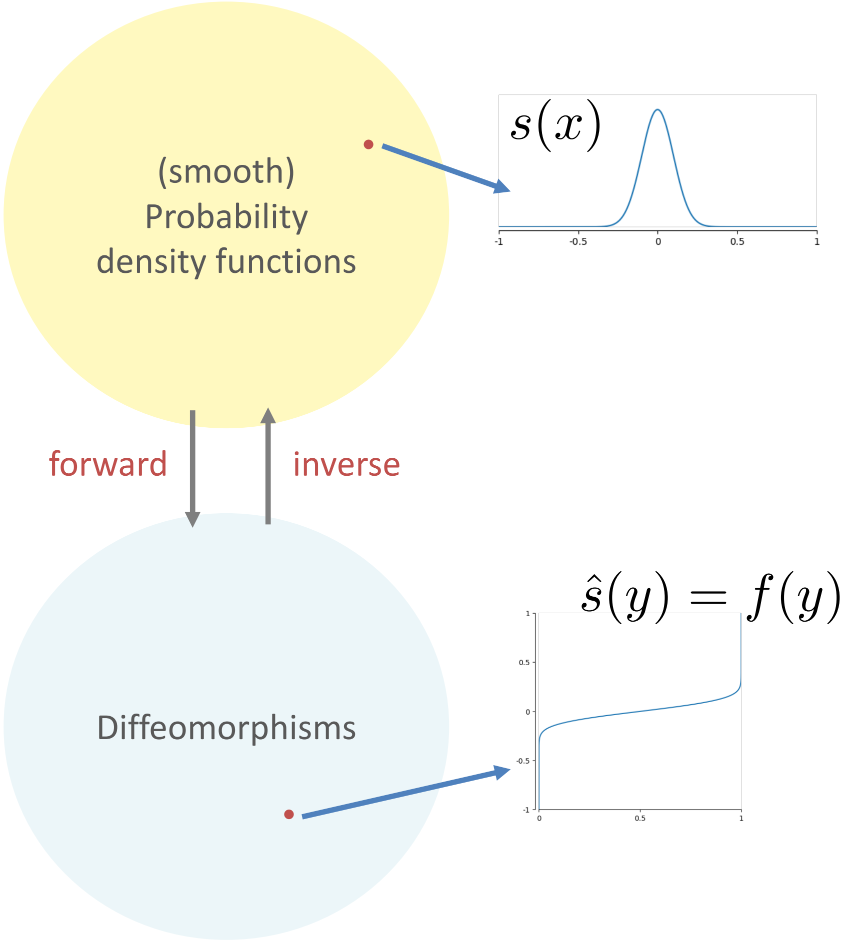

$$f : \Omega_0 \to \Omega$$

$$f'(y)s(f(y)) = s_0(y)$$

$$S(f(y)) = S_0(y)$$

$$y \in \Omega_0$$

$$\hat{s}(y) = f(y)$$



# 1-D Transport transform

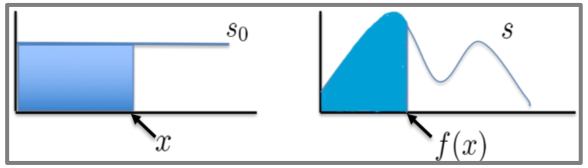
cumulative distribution transform (CDT)

[Park, Kolouri, Kundu, Rohde, ACHA 2018]

Let $s(x), s_0(x) > 0$ and $\int s(x)dx = \int s_0(x)dx = 1$ (PDFs)

Consider a map $f(x)$ , such that

$$\underbrace{\int_{-\infty}^{x} s_0(u)du}_{} = \underbrace{\int_{-\infty}^{f(x)} s(u)du}_{}$$

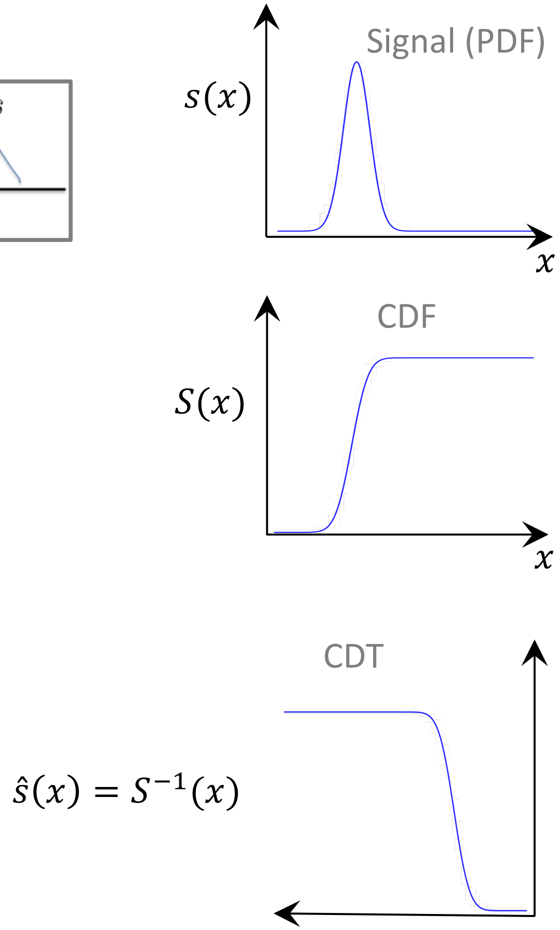



antiderivative

$$S_0(x) = S(f(x)) \implies f(x) = S^{-1}(S_0(x))$$

transform of $s$

Eg.: Let $s_0 = \chi_{[0,1]}$. Then, $S_0(x) = x$ and $f(x) = S^{-1}(x)$.




transform equations

$$\hat{s}(x) := S^{-1}(x) \quad \text{forward}$$

$$s(x) = \left(\hat{s}^{-1}(x)\right)' \quad \text{inverse}$$

# Extension to 2d, 3d: Radon Cumulative distribution transform (R-CDT)

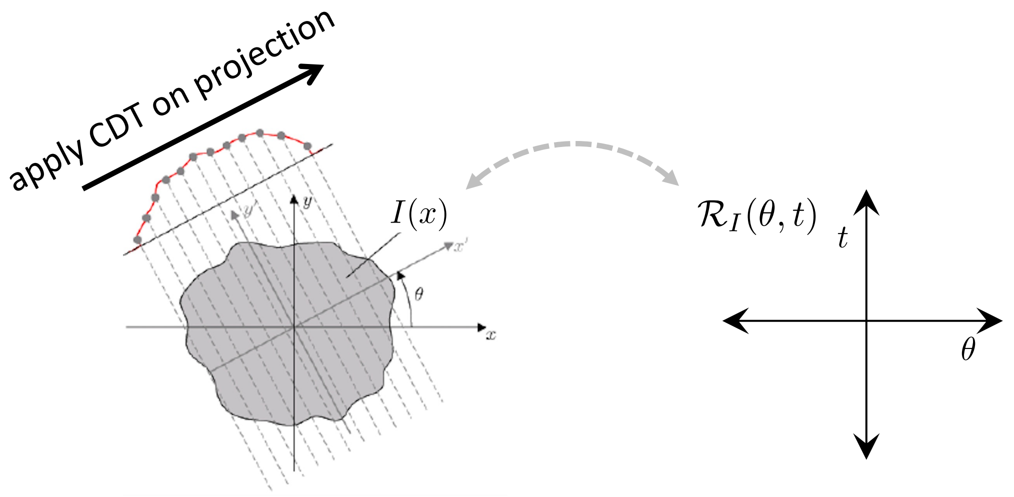
## Radon Transform

Forward

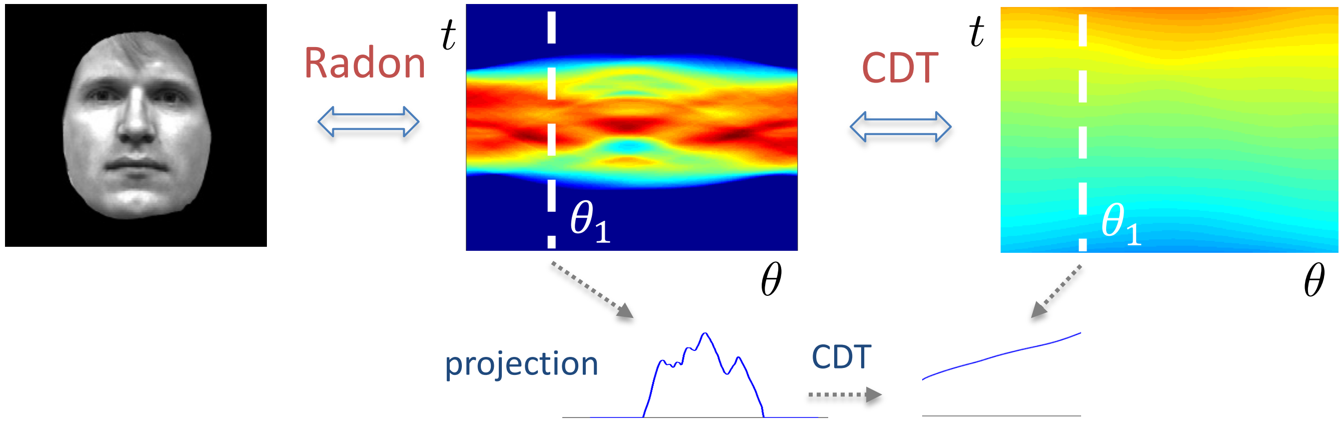
$$\mathcal{R}_I(\theta, t) = \int I(x)\delta(t - \omega \cdot x)dx$$
$$\omega = [\sin\theta, \cos\theta]^T$$

Inverse

$$\mathcal{R}_I^{-1}(x) = \int_{\mathbb{S}^{d-1}} (\mathcal{R}I(\cdot, \theta) * \eta(\cdot)) \circ (x \cdot \theta) d\theta$$




## Radon Cumulative Distribution Transform




[Kolouri, Park, Rohde, IEEE TIP 2016]

# CDT Properties

## Translation

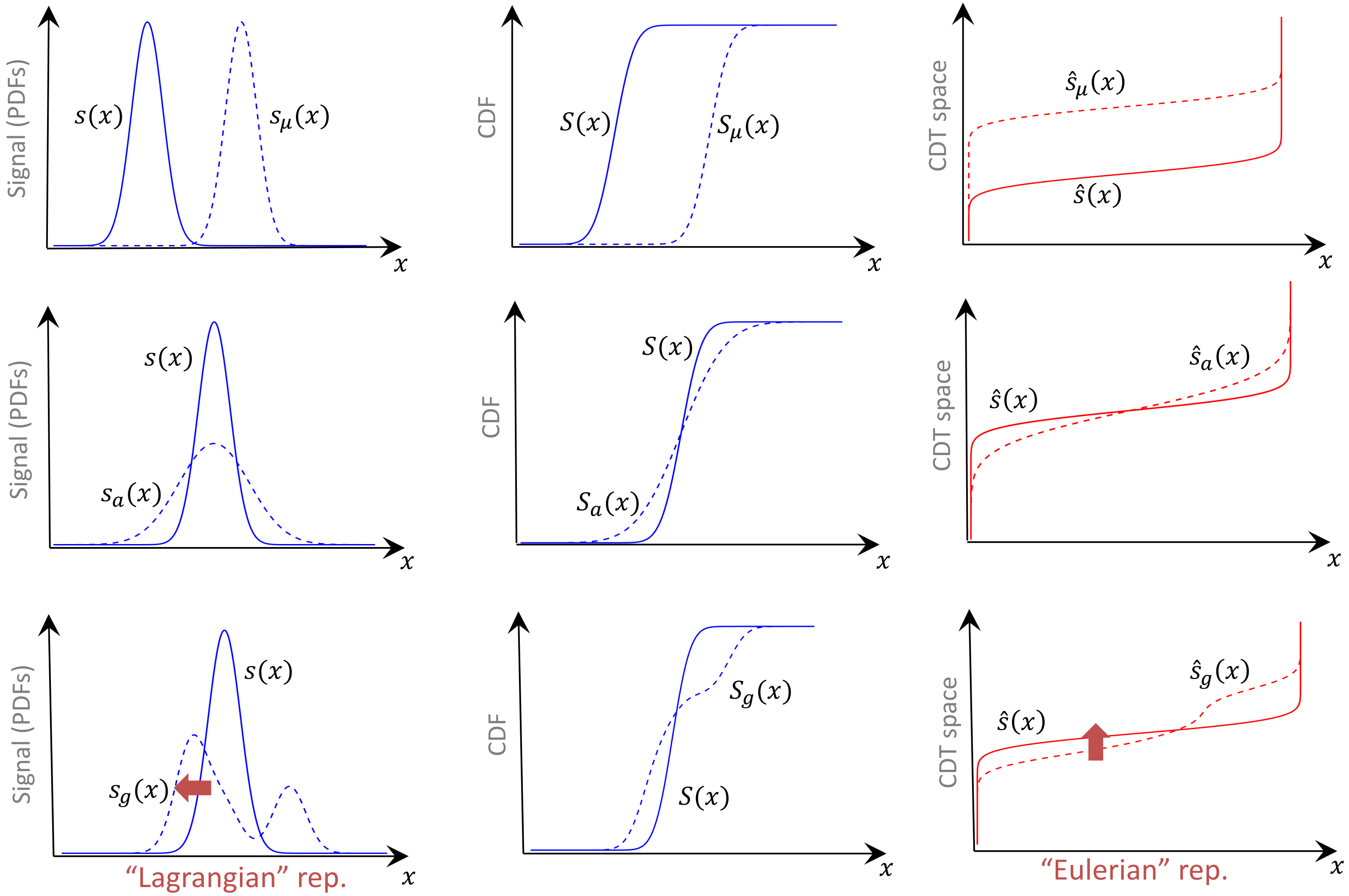
$$s_\mu(x) = s(x-\mu)$$
$$\hat{s}_\mu(x) = \hat{s}(x) + \mu$$

## Scaling

$$s_a(x) = as(ax)$$
$$\hat{s}_a(x) = \frac{\hat{s}(x)}{a}$$

## Composition

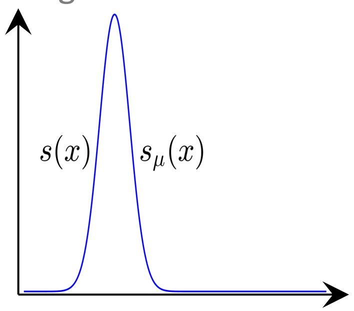
$$s_g(x) = g'(x)s(g(x))$$
$$\hat{s}_g(x) = g^{-1}(\hat{s}(x))$$



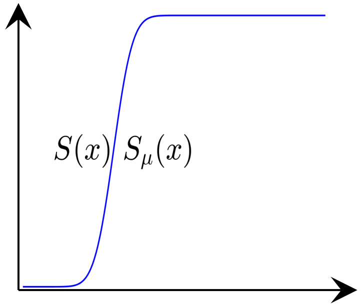
# Data Geometry

Consider the data set generated by a signal and all of its translates

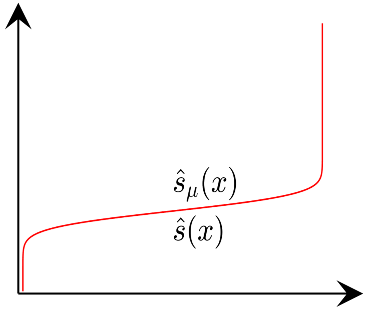
Signal



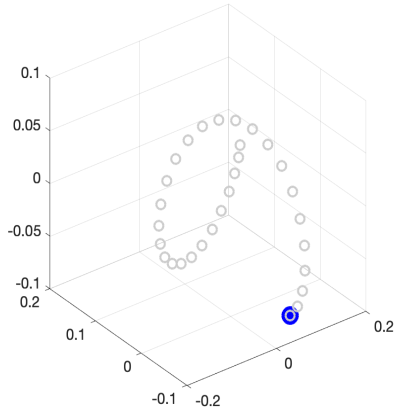

CDF



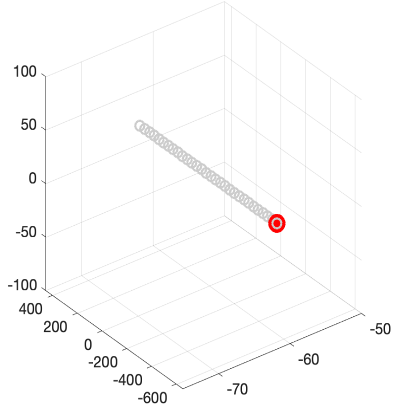

CDT




PCA (3 components) of dataset




nonlinear/nonconvex




linear/convex

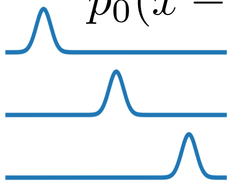
# Signal classes: generative model

## signal space

Class $\mathbb{P}$

template 1 $p_0$

$p_0(x-\mu)$

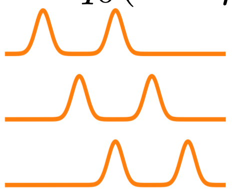


$$\mathbb{P} = \{p_g | p_g(x) = g'(x) p_0(g(x))\}$$

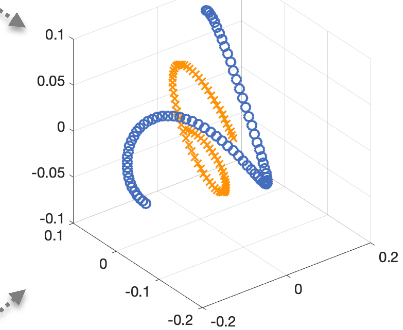
Class $\mathbb{Q}$

template 2 $q_0$

$q_0(x-\mu)$



$$\mathbb{Q} = \{q_g \, | q_g(x) = g'(x) q_0(g(x))\}$$



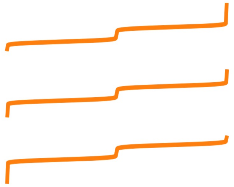

low dimensional projection

## transform space

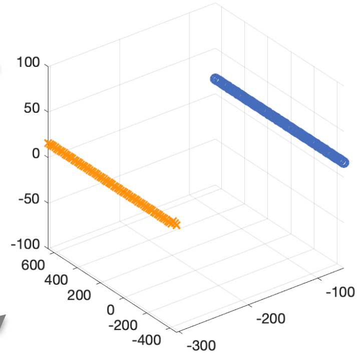
$\hat{\mathbb{P}}$

$\hat{p_g} = \hat{p} + \mu$

$\hat{\mathbb{Q}}$



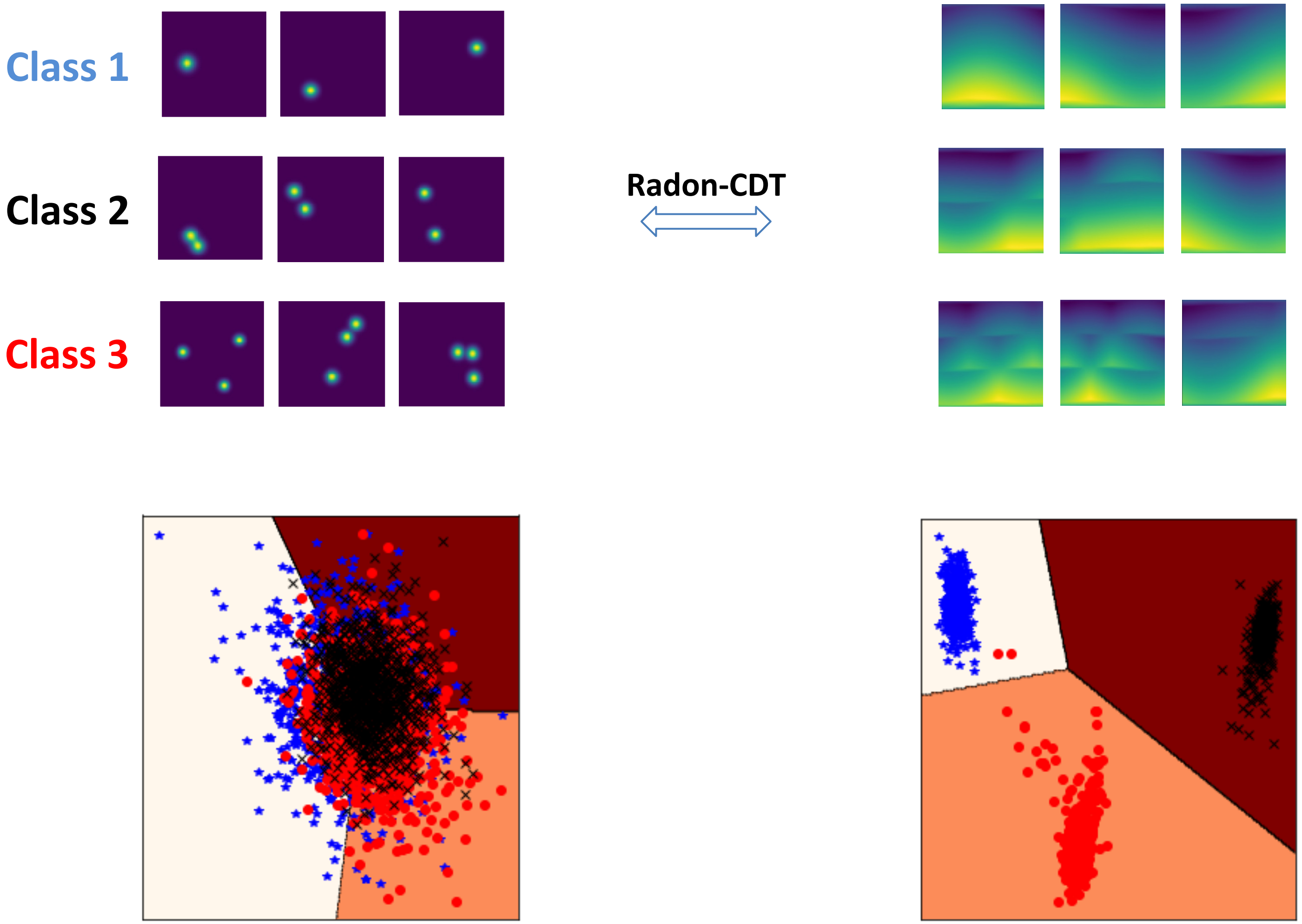
$\hat{q}_g = \hat{q} + \mu$




low dimensional projection

"confound" $g \in \mathcal{C}$ e.g.: translation $g(x) = x - \mu$

# 2D Example




LDA subspace of original data

LDA subspace of transformed data

# CDT: linear separation theorem

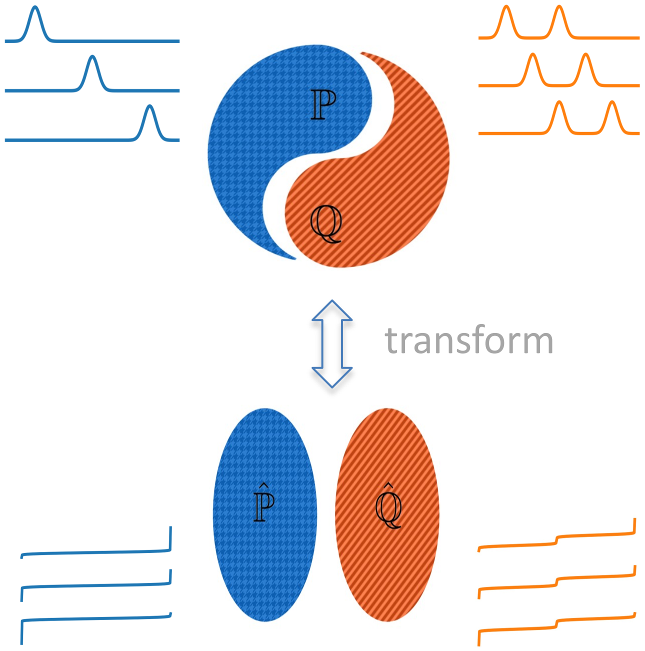
Consider signal classes:

$\mathbb{P} = \{p_g : g \in \mathcal{C}\}$ and $\mathbb{Q} = \{q_g : g \in \mathcal{C}\}$

where $p_g(x) = g'(x)p_0(g(x))$

$q_g(x) = g'(x)q_0(g(x))$

$\mathbb{P} \cap \mathbb{Q} = \emptyset$




Theorem:

$\hat{\mathbb{P}}$, $\hat{\mathbb{Q}}$ will be *convex*

iff $\mathcal{C}^{-1}$ is convex.

for any reference $s_0$

does not require knowledge of $p_0$ $q_0$

Corollary: $\hat{\mathbb{P}}$, $\hat{\mathbb{Q}}$ linearly separable

if $\mathcal{C}^{-1}$ is convex.

[Park, Kolouri, Kundu, Rohde, ACHA 18]
[Aldroubi, Li, Rohde, SaSiDa, 21]
[Shifat-E-Rabbi, …, Rohde, JMIV 21]
[Aldroubi et al, AIMS Foundations of Data Science, 22]

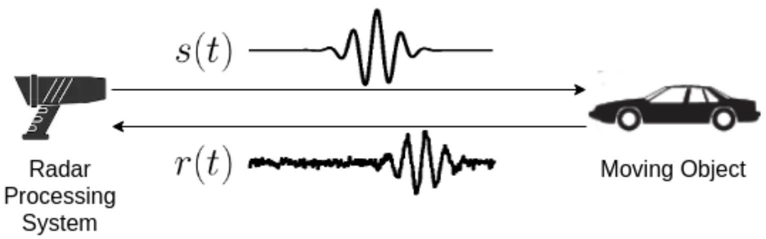
To see this: $\widehat{\mathbb{P}} = \{g^{-1}(\hat{p}(y)) | g^{-1} \in \mathcal{C}^{-1}\}$

$\widehat{\mathbb{Q}} = \{g^{-1}(\hat{q}(y)) | g^{-1} \in \mathcal{C}^{-1}\}$

$\Longrightarrow$ convex iff $\mathcal{C}^{-1}$ convex

# Applications

# Parametric signal estimation




Static object: $r(t) = s(t - \tau) + \eta(t)$

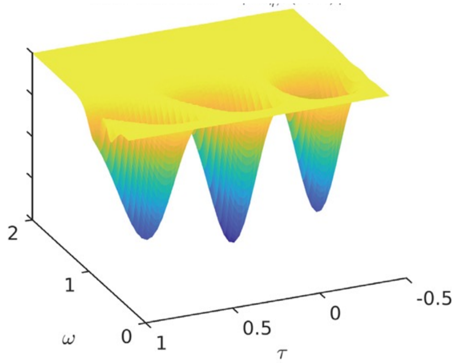
Moving object: $r(t) = s(\omega t - \tau) + \eta(t)$

$g_p(t) = \omega t - \tau$

## Traditional approach:

$$A_{r,s}(\tau, \omega) = \sqrt{\omega} \int r(t) s(\omega t - \tau) dt$$

$$\arg\min_{\tau,\omega} -|A_{r,s}(\tau, \omega)|$$

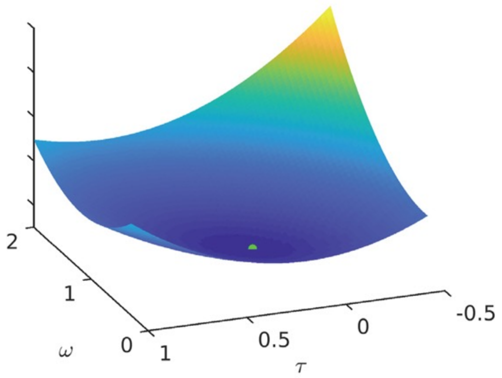



→ computationally expensive
global optimization required

## Wasserstein minimization

$$W^2(r, s) = \|\omega \hat{r} - \tau - \hat{s}\|^2$$

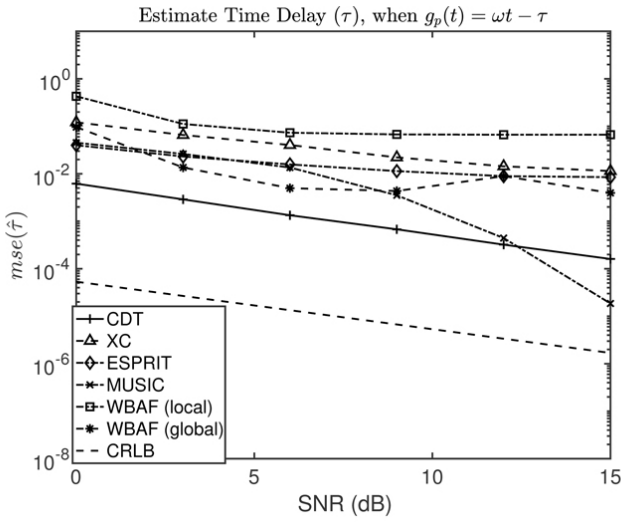
$$\arg\min_{\tau,\omega} \|\omega \hat{r} - \tau - \hat{s}\|^2$$

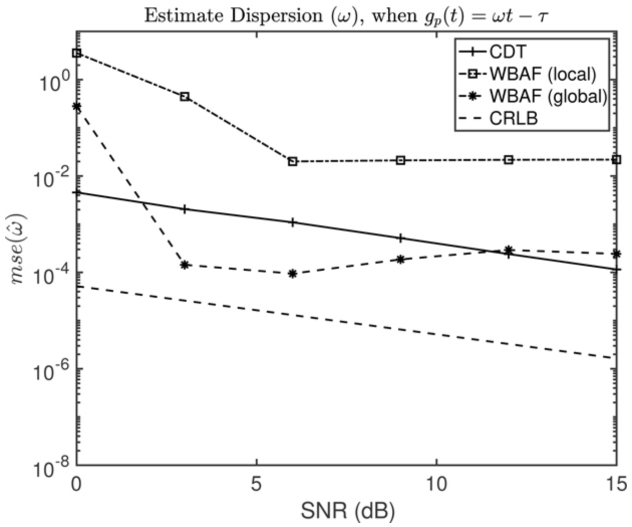


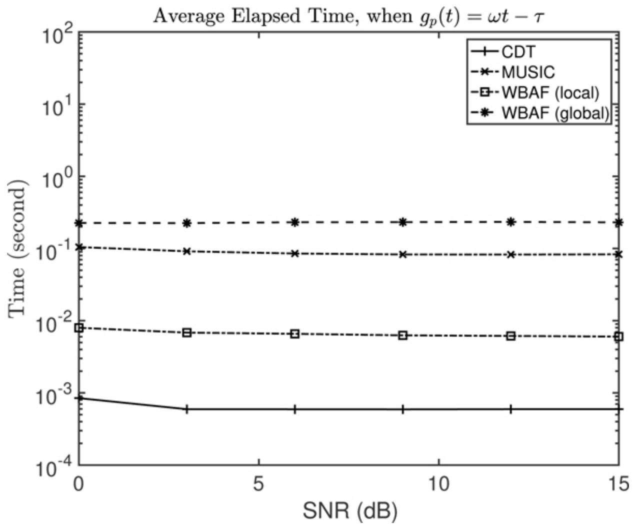

→ linear least squares
cheap to compute

AHM Rubaiyat, KM Hallam, JM Nichols, MN Hutchinson, S Li, GK Rohde, Parametric Signal Estimation Using the Cumulative Distribution Transform, IEEE Trans. Signal Processing, 68, pp. 3312-3324, 2020.

# Parametric signal estimation



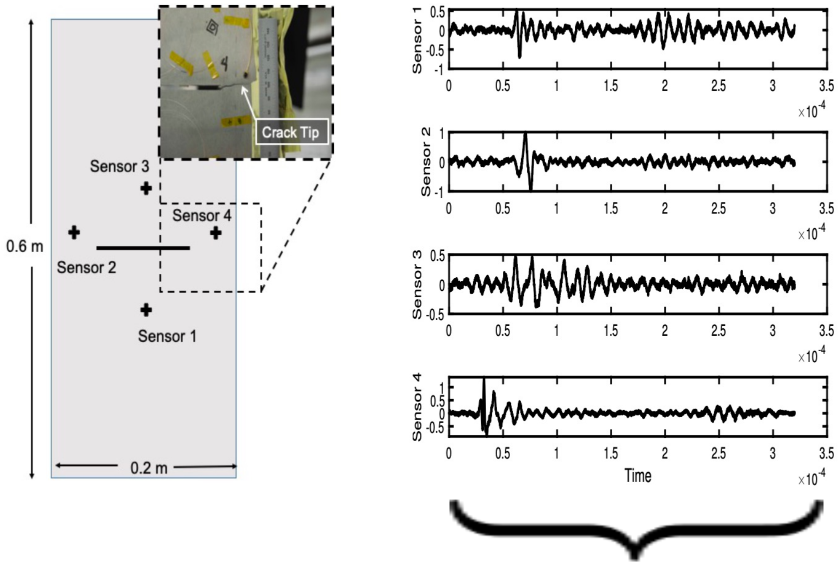




orders of magnitude cheaper to compute

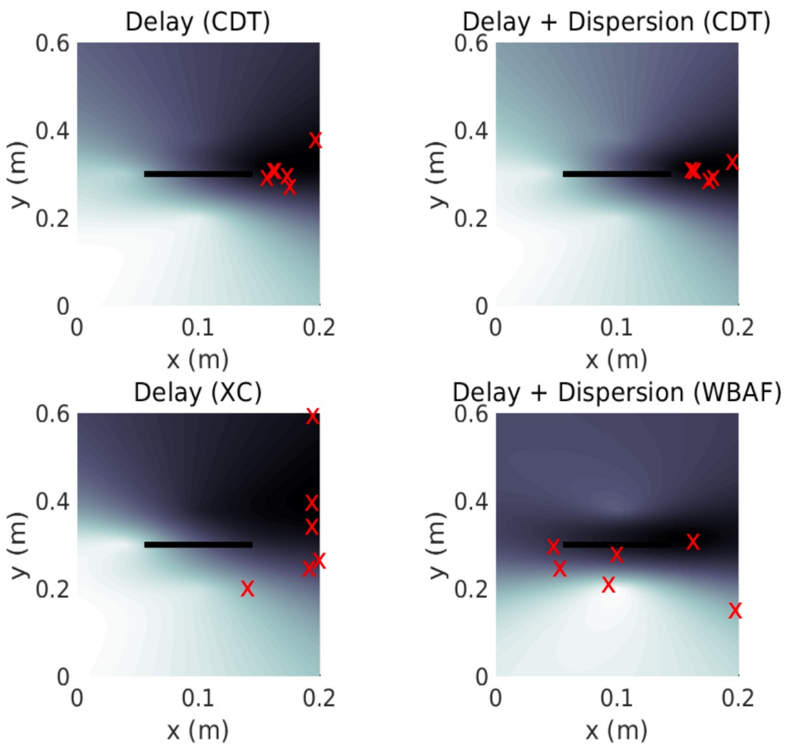
AHM Rubaiyat, KM Hallam, JM Nichols, MN Hutchinson, S Li, GK Rohde, Parametric Signal Estimation Using the Cumulative Distribution Transform, IEEE Trans. Signal Processing, 68, pp. 3312-3324, 2020.

# Parametric signal estimation in structural health monitoring




Estimate TDOA in CDT domain

Crack localization

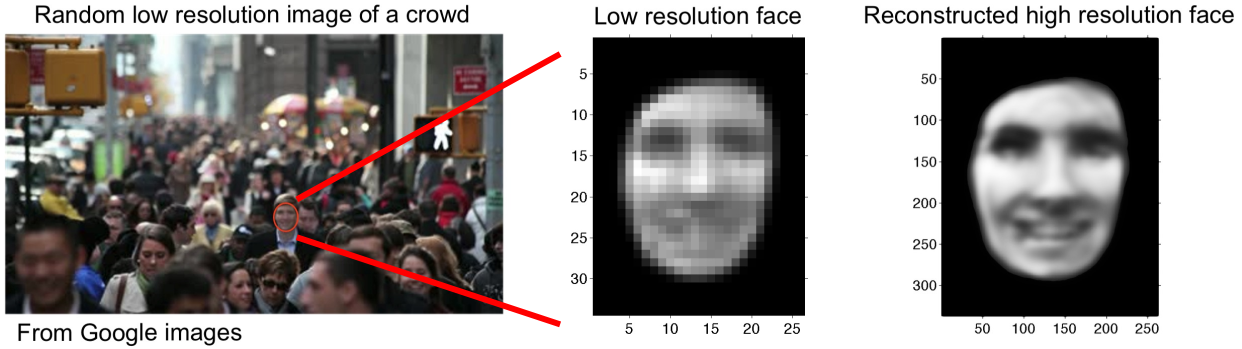



AHM Rubaiyat, KM Hallam, JM Nichols, MN Hutchinson, S Li, GK Rohde, Parametric Signal Estimation Using the Cumulative Distribution Transform, IEEE Trans. Signal Processing, 68, pp. 3312-3324, 2020.

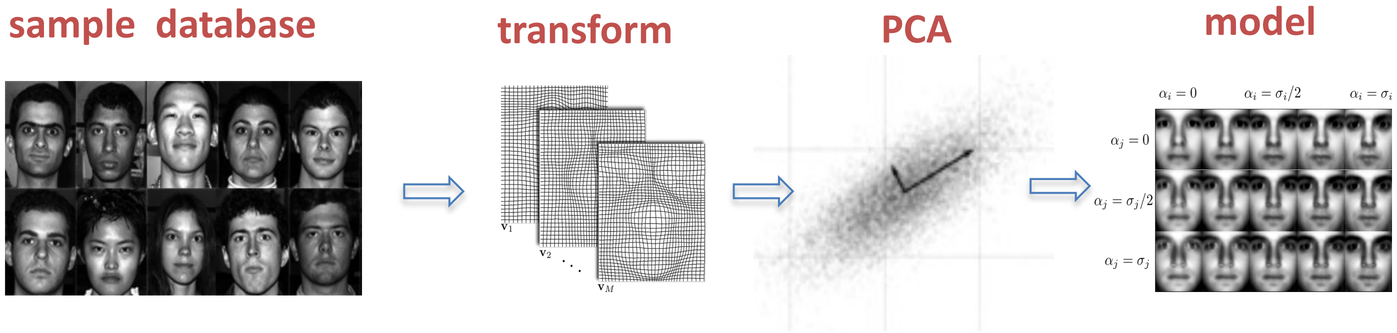
# Model-based inverse problems

## Enhancing an image

### Find model parameters that best match data




## Modeling phase



# Application: single image super resolution

[Kolouri, Rohde, CVPR 2015]

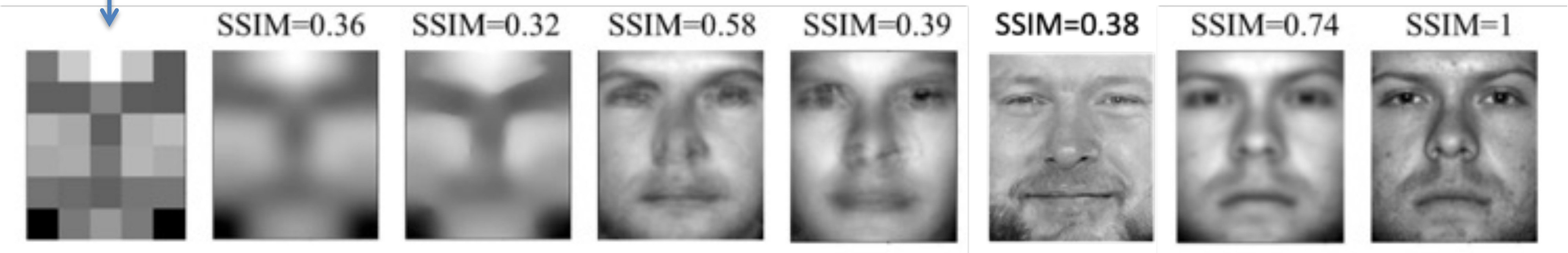
**Model:** $M_\alpha = D_{f_\alpha}(x) I_0(f_\alpha(x))$ $\quad f_\alpha(x) = \underbrace{\sum_k \alpha_k \phi_k(x)}_{\text{Transport PCA trained on high res database}}$

low-res data

**Fit:** $\min_\alpha \|I - V M_\alpha\|^2$

degradation

5x6 pixels!




Input low-res data | B-spline interp. | compressed sensing | K-means nonlinear mapping | Kernel PCA | PULSE generative modeling | transport method | ground truth

# Applications: classification

# Problem statement (supervised learning)

Training: given sample signals/images from different classes

$$\{s_1^{(1)}, s_2^{(1)}, \cdots\} \qquad s_j^{(k)} \in \mathbb{S}^{(k)} = \left\{ s_j^{(k)} | s_j^{(k)} = |\det J g_j| \varphi^{(k)} \circ g_j, \forall g_j \in \mathcal{G} \right\}$$

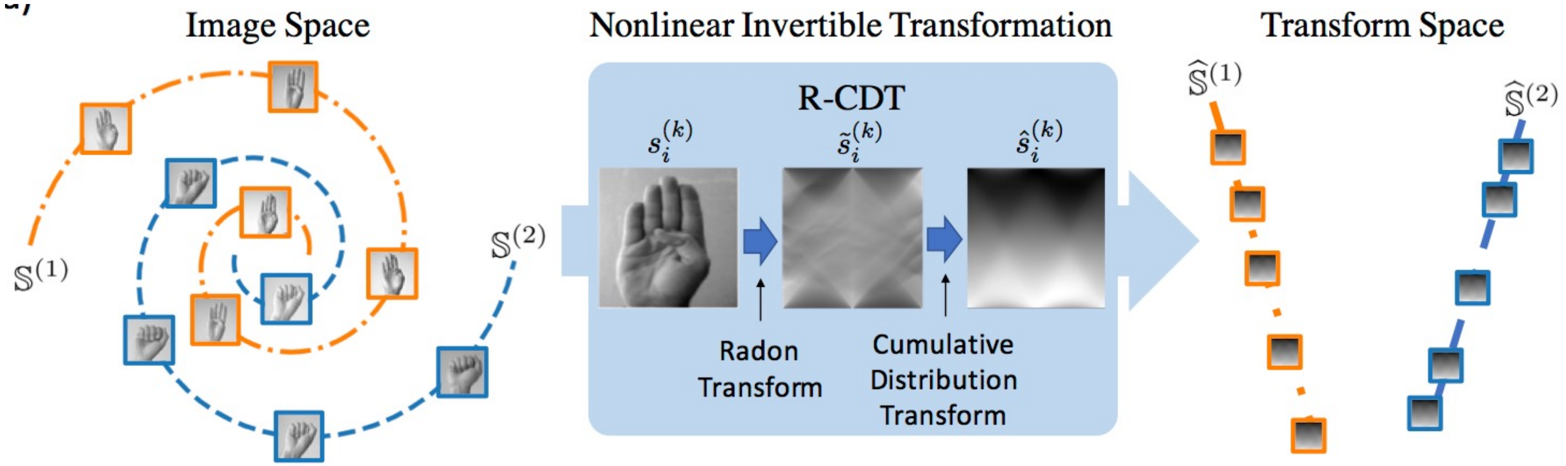
$$\{s_1^{(2)}, s_2^{(2)}, \cdots\}$$

$\varphi^{(k)}$ unknown

$g_j \in \mathcal{G}$ unknown

Testing: find the class that a test sample $s$ originated from.

# Solution overview



# RCDT – nearest subspace

**Training:** estimating basis vectors for subspaces corresponding to each class

training data $\left\{s_1^{(k)}, s_2^{(k)}, \cdots\right\}$ ⇨ transform $\left\{\widehat{s}_1^{(k)}, \widehat{s}_2^{(k)}, \cdots\right\}$ ⇨ deformation model $\left\{\widehat{s}_1^{(k)}, \widehat{s}_2^{(k)}, \cdots\right\} \cup \mathbb{U}_T$ ⇨ orthogonalize $\left[\widehat{b}_1^{(k)}, \widehat{b}_2^{(k)}, \cdots\right] = B^{(k)}$ ⇨ output $B^{(k)}$

**Testing :** predicting the test sample as belonging to the class corresponding to the nearest subspace

test sample $s$ ⇨ transform $\hat{s}$ ⇨ nearest subspace $\arg\min_k ||\widehat{s} - A^{(k)}\widehat{s}||^2$ → class 1, class 2, ⋮

Here, $A^{(k)} = B^{(k)} B^{(k)T}$

Translation $\mathbb{U}_T = \{u_T^{(1)}(t,\theta), u_T^{(2)}(t,\theta)\}$, with $u_T^{(1)}(t,\theta) = \cos\theta$ and $u_T^{(2)}(t,\theta) = \sin\theta$

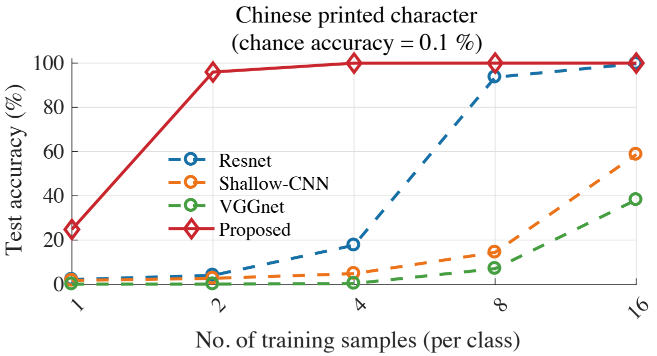
Isotropic scaling $\mathbb{U}_D = \varnothing$

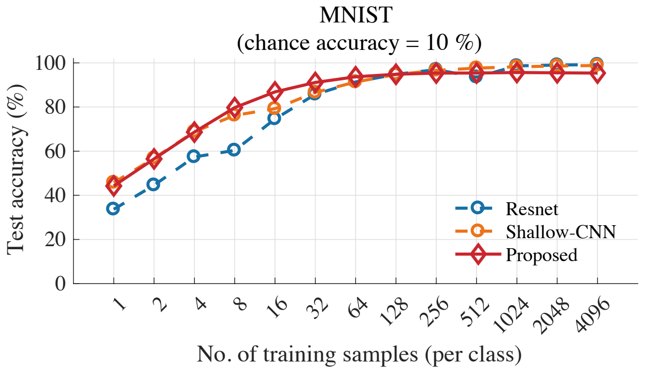
Two important aspects of the method:

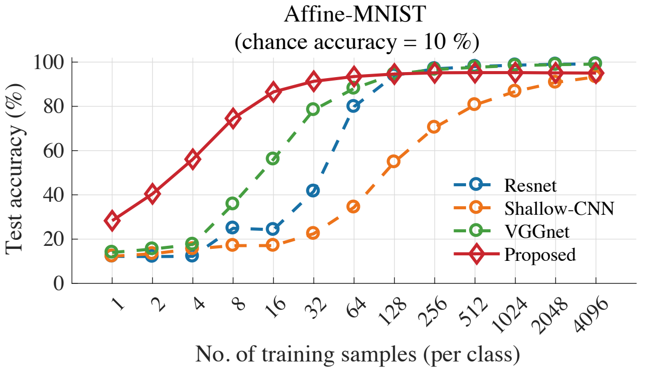
1. The method can learn from datasets
2. The method can learn from mathematically prescribed sets without explicitly using the training data

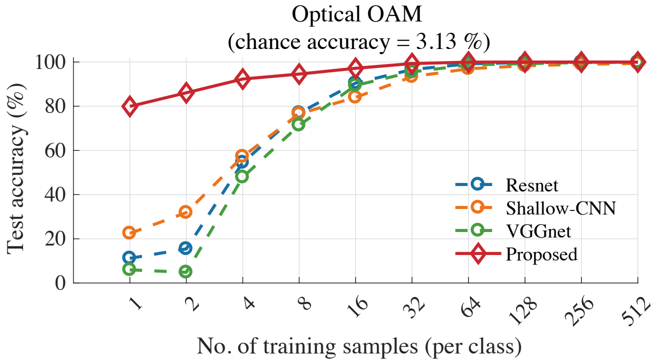
# Results and comparisons with deep learning

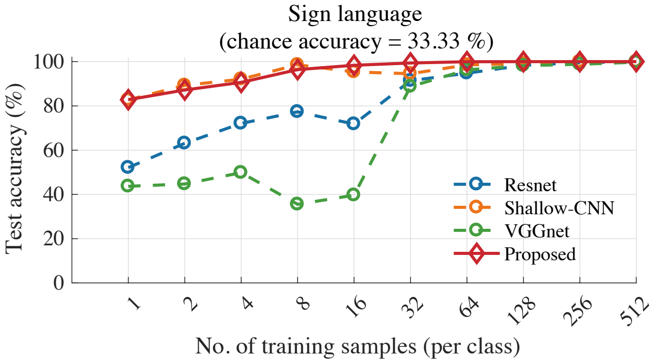
[Shifat-E-Rabbi et al, JMIV 2021]

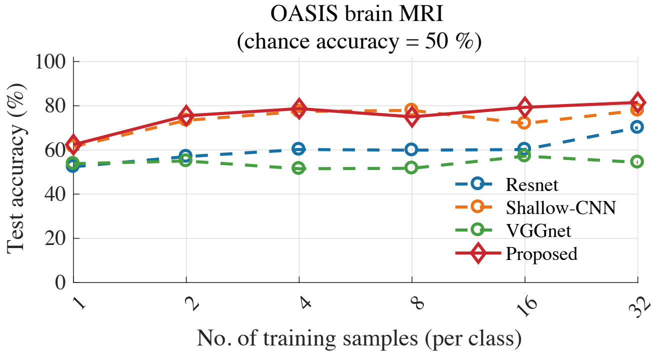


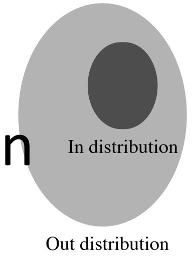



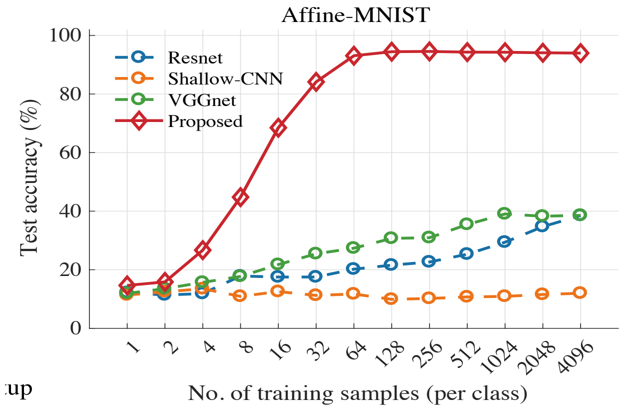



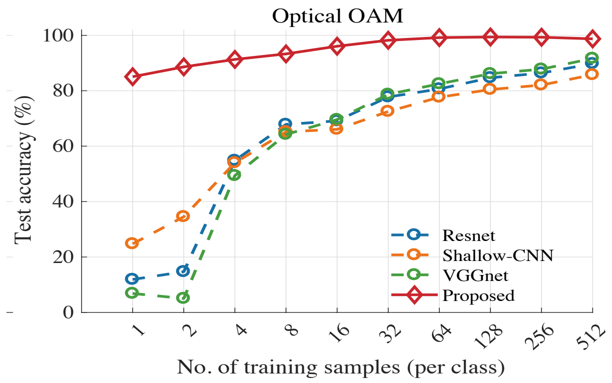



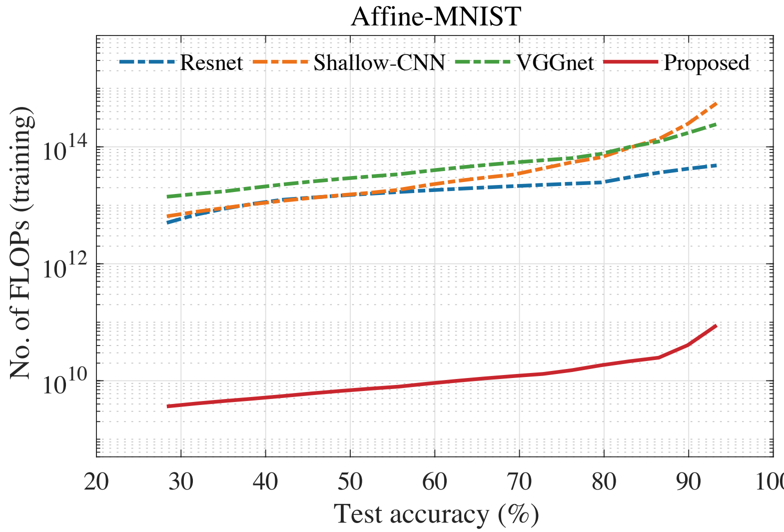

## Out of distribution testing

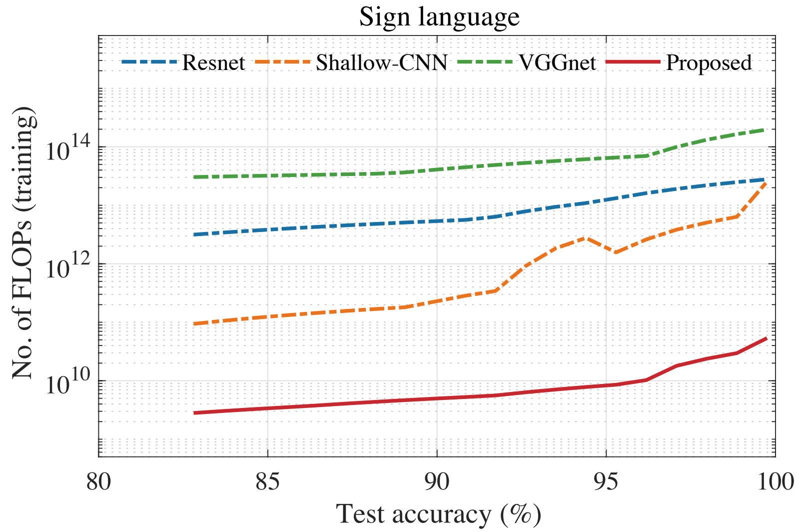


# Results and comparisons with deep learning



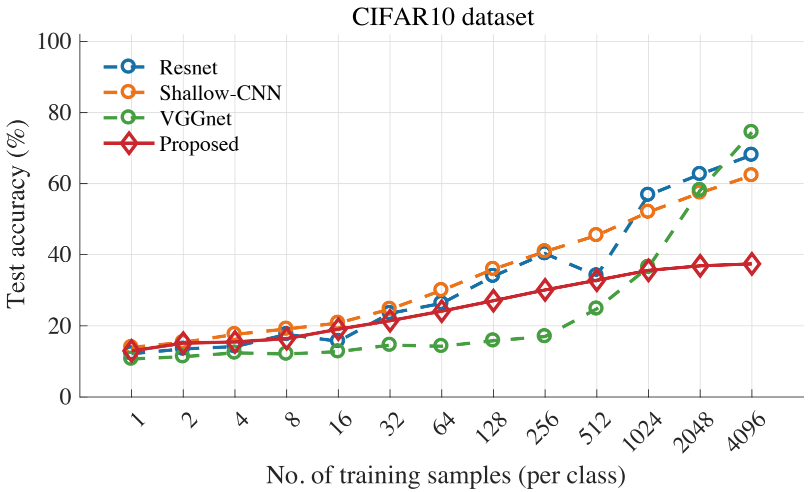

## Limitations

$$\mathbb{P} = \{p_g : g \in \mathcal{C}\} \quad \mathbb{Q} = \{q_g : g \in \mathcal{C}\}$$

where

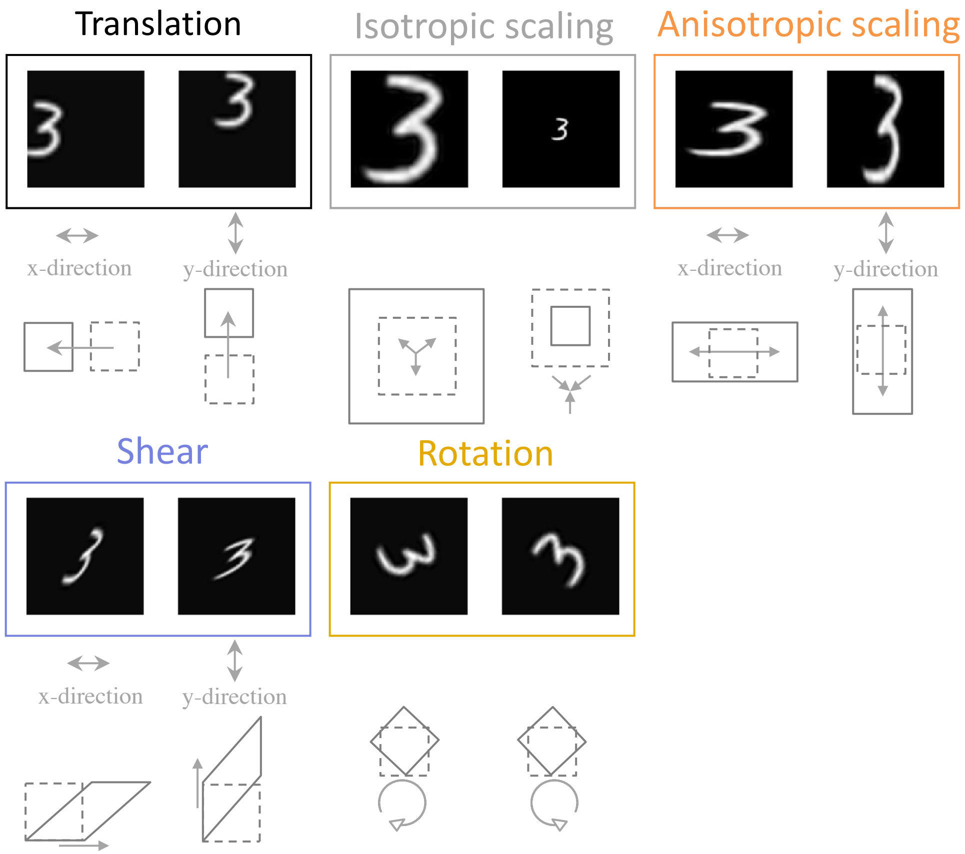
$$p_g(x) = g'(x)p_0(g(x))$$
$$q_g(x) = g'(x)q_0(g(x))$$

It doesn't seem to work as well for general object detection



# Invariance encoding

Aim: Can we mathematically model invariances in data?

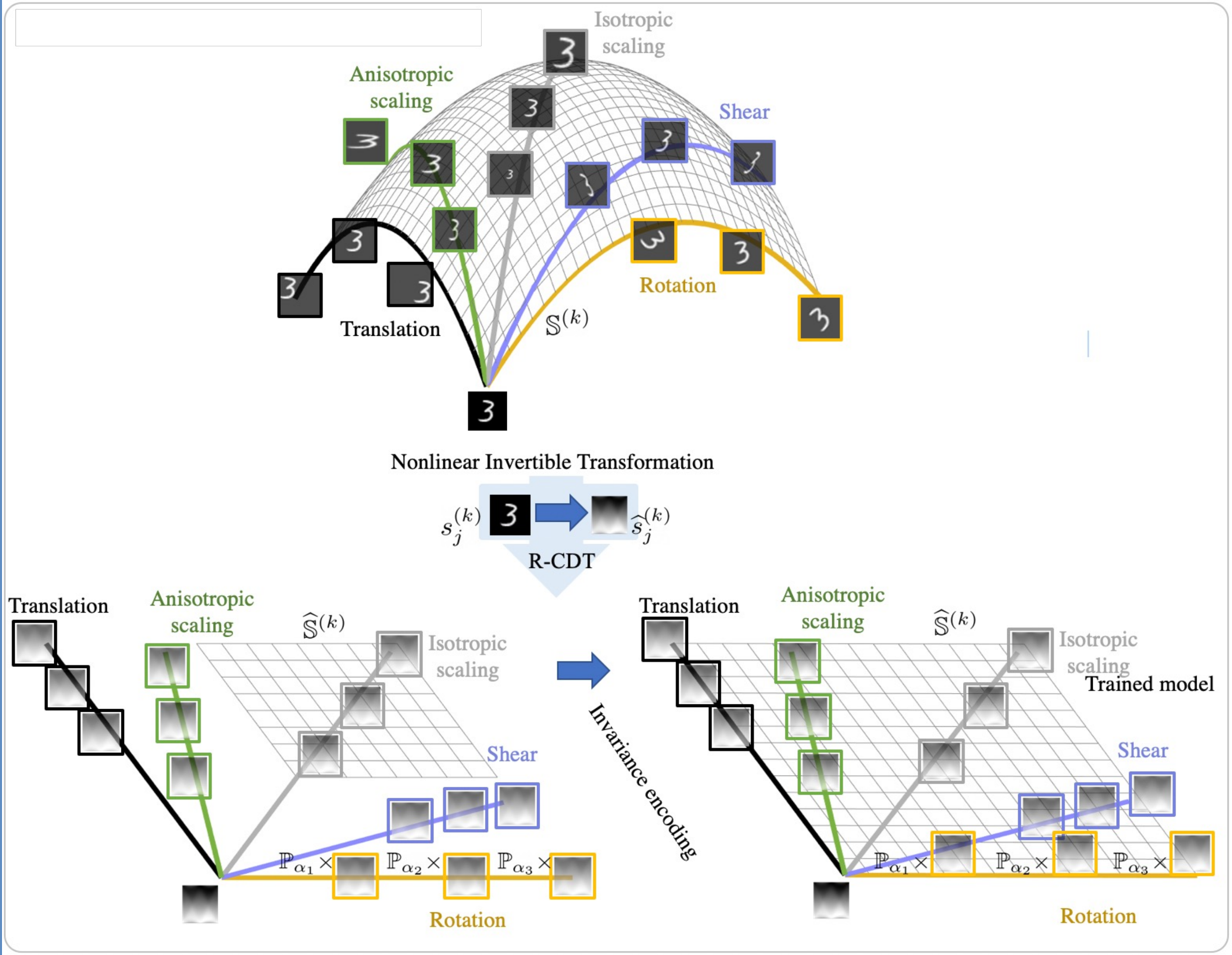



$$\{s_1^{(1)}, s_2^{(1)}, \cdots\} \quad \{s_1^{(2)}, s_2^{(2)}, \cdots\}$$

$$s_j^{(k)} \in \mathbb{S}^{(k)} = \left\{ s_j^{(k)} | s_j^{(k)} = |\det J g_j| \varphi^{(k)} \circ g_j, \forall g_j \in \mathcal{G} \right\}$$

$\varphi^{(k)}$ unknown $\qquad g_j \in \mathcal{G}$ unknown + affine

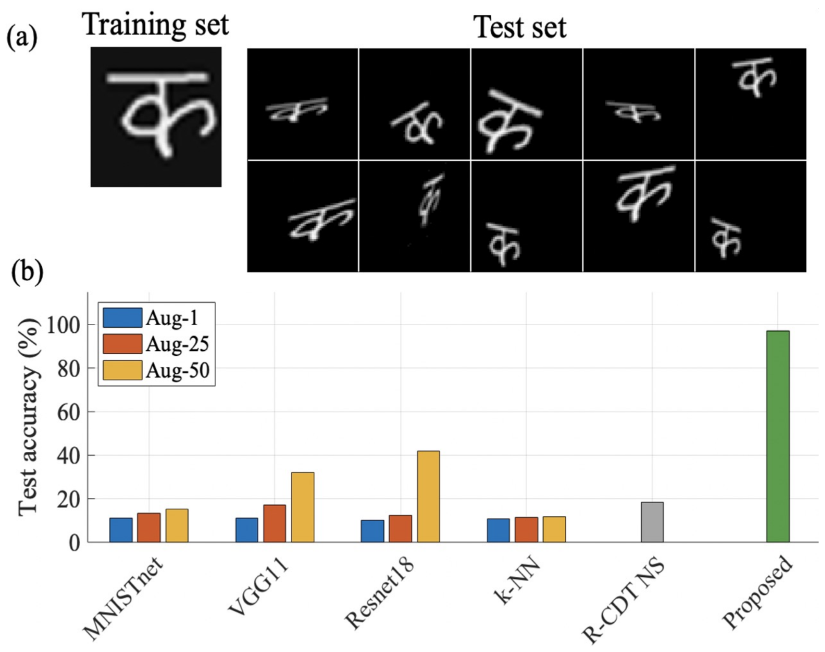
Solution overview:



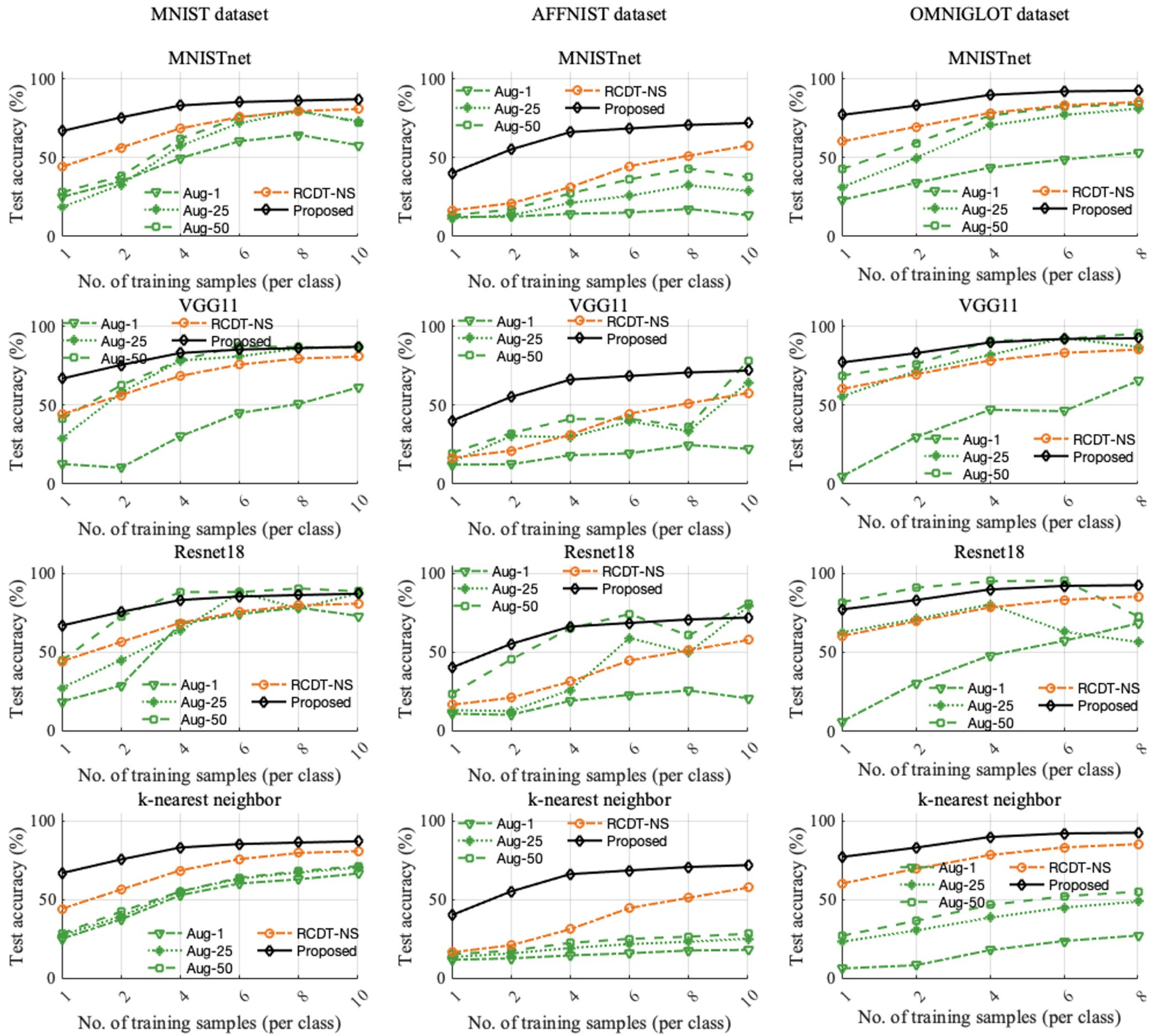
# Results and comparisons with deep learning

[Shifat-E-Rabbi et al, arxiv 2022]

## Synthetic data



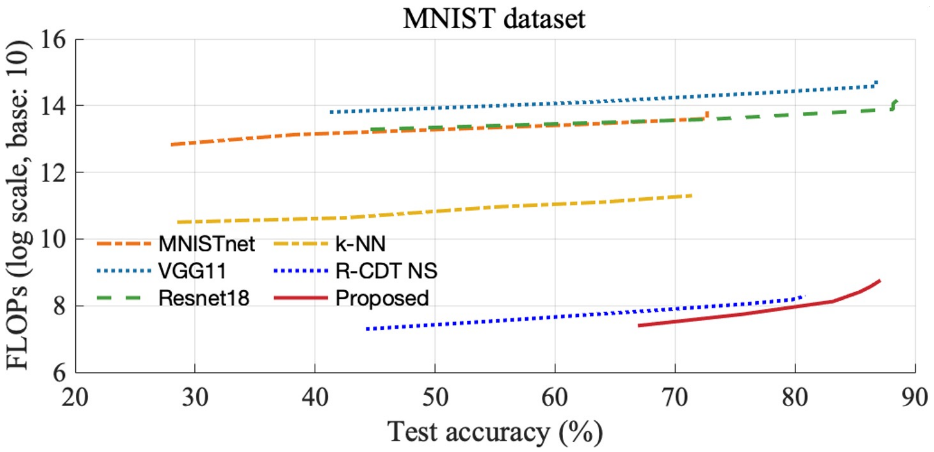

## Real data



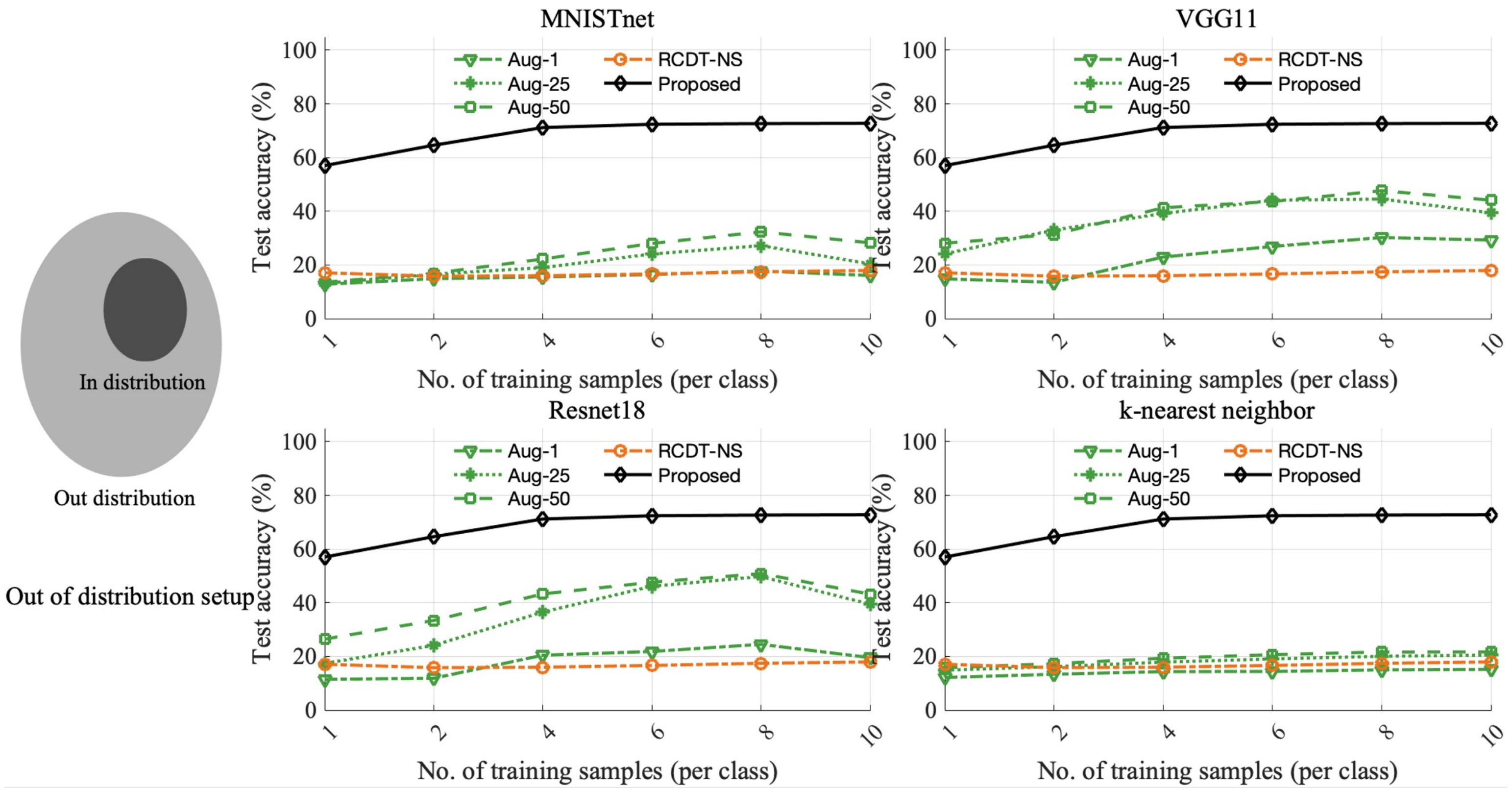
# Results and comparisons with deep learning

[Shifat-E-Rabbi et al, arxiv 2022]

## Computational complexity

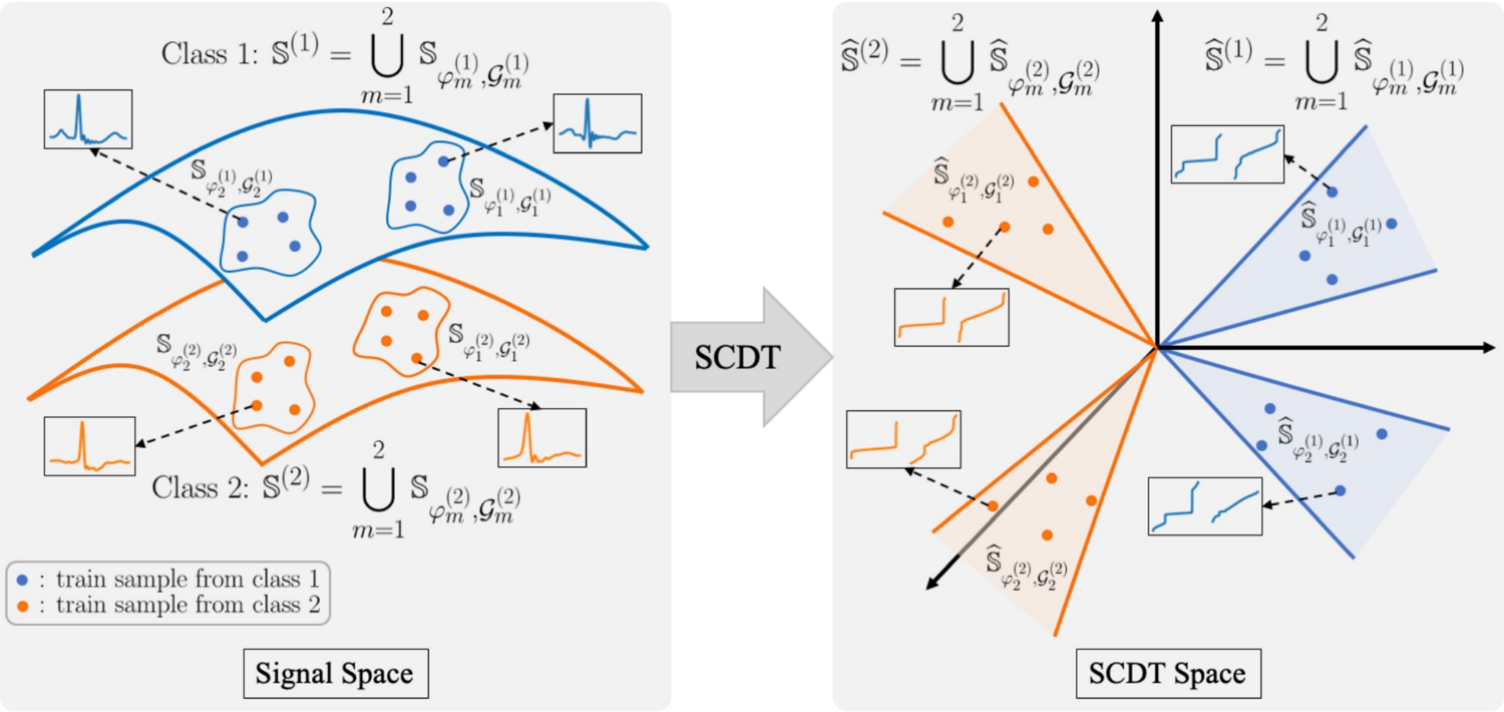



## Out-of-distribution experiment



# End-to-End Time Series Classification [Rubaiyat et al, ArXiv 2022]

Generative Model:




$$\mathbb{S}^{(c)} = \bigcup_{m=1}^{M_c} \mathbb{S}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}},$$

$$\mathbb{S}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}} = \left\{ s_{j,m}^{(c)} | s_{j,m}^{(c)} = g_j' \varphi_m^{(c)} \circ g_j, g_j' > 0, g_j \in \mathcal{G}_m^{(c)} \right\},$$

$$\left(\mathcal{G}_m^{(c)}\right)^{-1} = \left\{ \sum_{i=1}^{k} \alpha_i f_{i,m}^{(c)}, \alpha_i \geq 0 \right\}$$

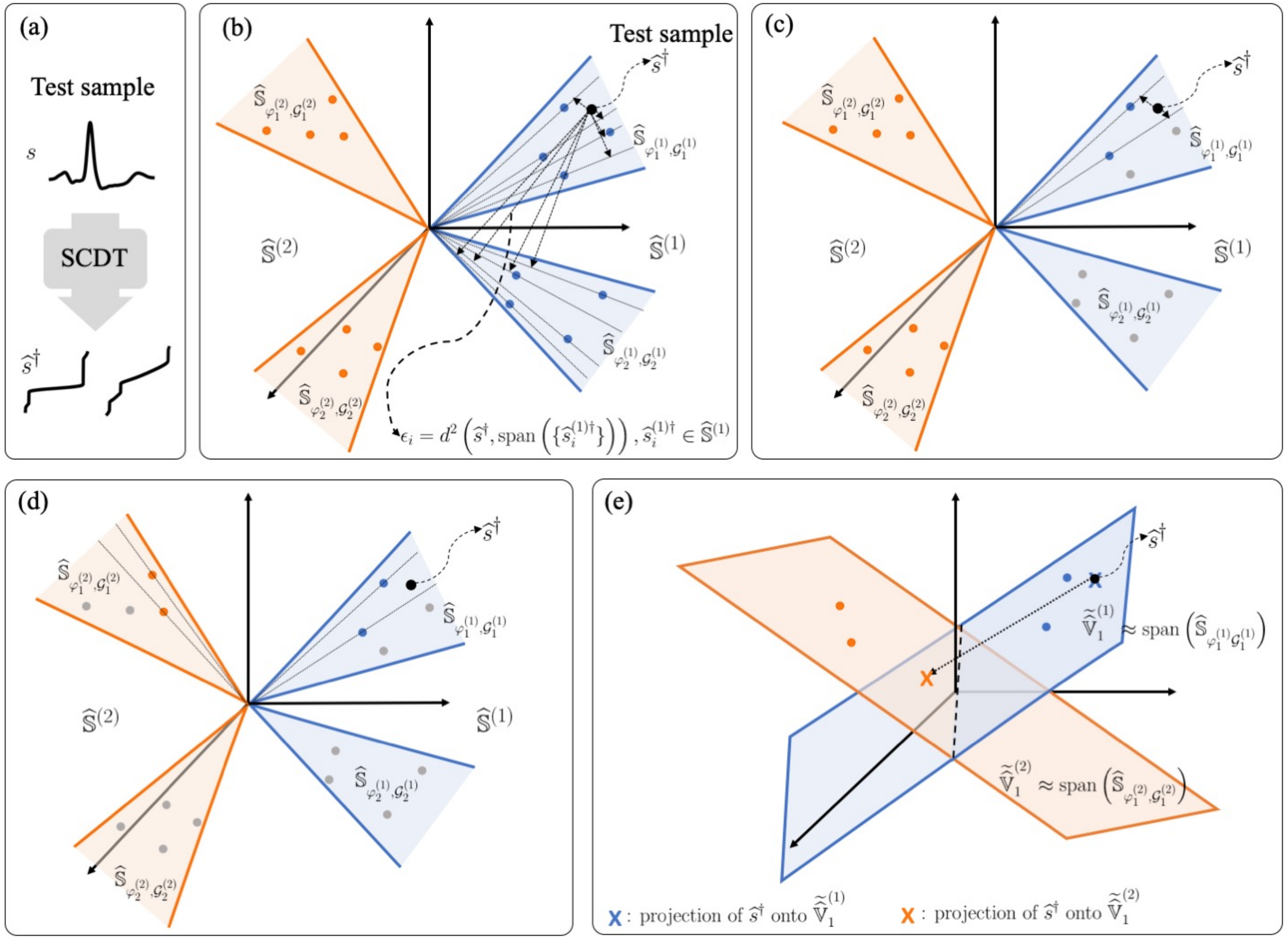
$$\widehat{\mathbb{S}}^{(c)} = \bigcup_{m=1}^{M_c} \widehat{\mathbb{S}}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}},$$

$$\widehat{\mathbb{S}}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}} = \left\{ \widehat{s}_{j,m}^{(c)} | \widehat{s}_{j,m}^{(c)} = g_j^{-1} \circ \widehat{\varphi}_m^{(c)}, g_j \in \mathcal{G}_m^{(c)} \right\},$$

$$\left(\mathcal{G}_m^{(c)}\right)^{-1} = \left\{ \sum_{i=1}^{k} \alpha_i f_{i,m}^{(c)}, \alpha_i \geq 0 \right\}$$

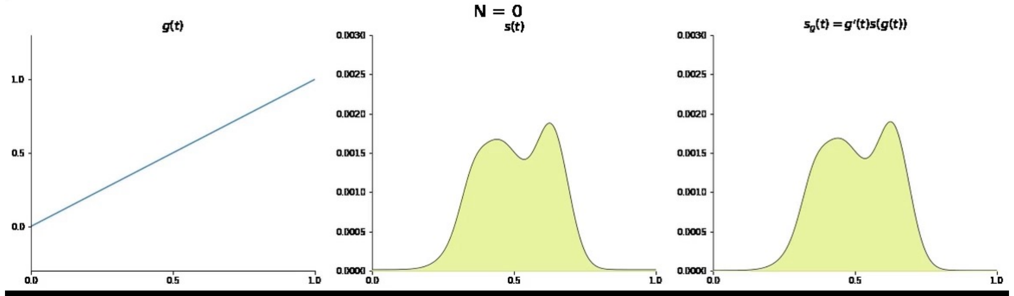
# End-to-End Time Series Classification [Rubaiyat et al, ArXiv 2022]

## NLS algorithm:

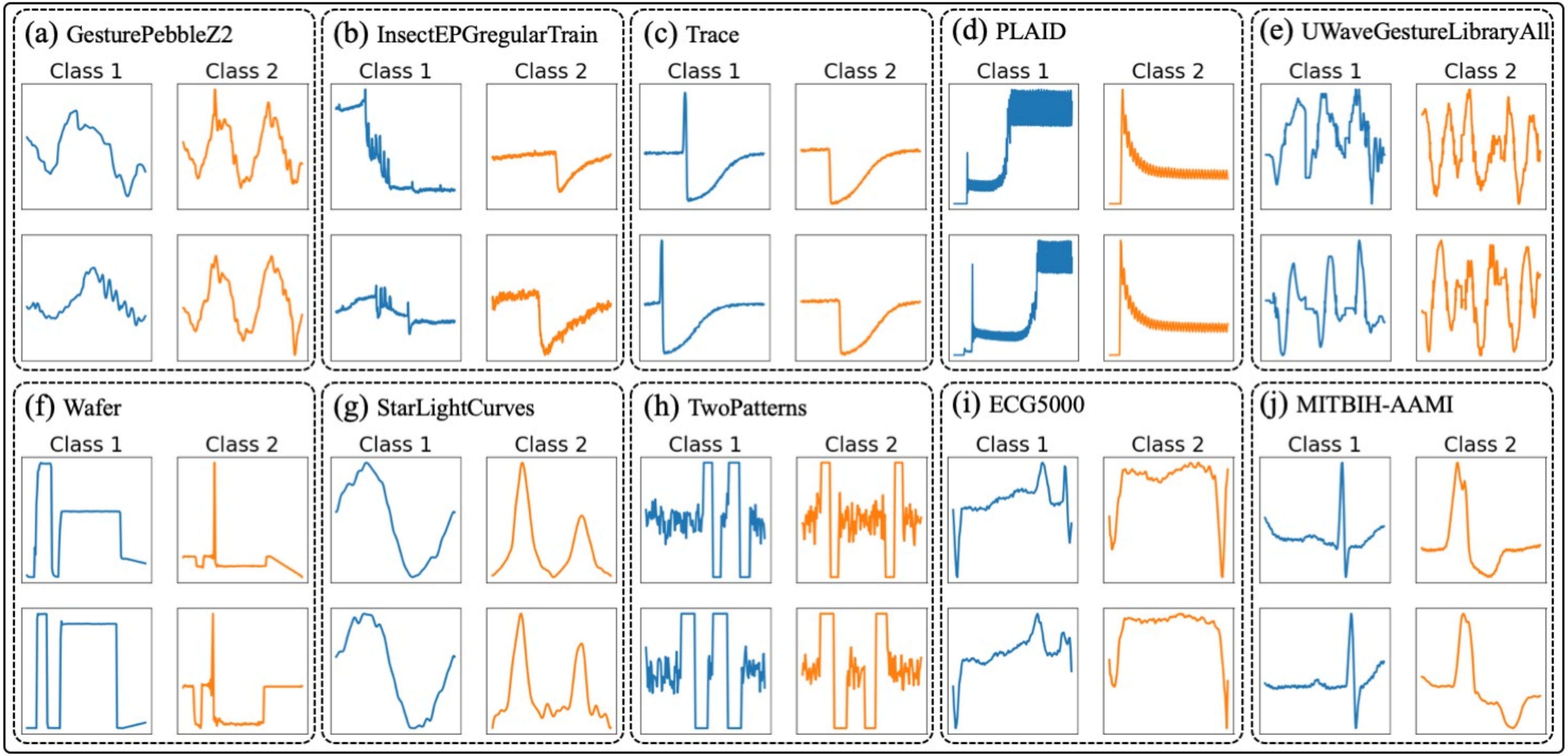



## Subspace Enrichment:

$$\widetilde{\widehat{\mathbb{V}}}_m^{(c)} = \text{span}\left(\{\widehat{z}_1^{(c)\dagger}, \cdots, \widehat{z}_k^{(c)\dagger}\} \cup \mathbb{U}_T \cup \mathbb{U}_H\right)$$



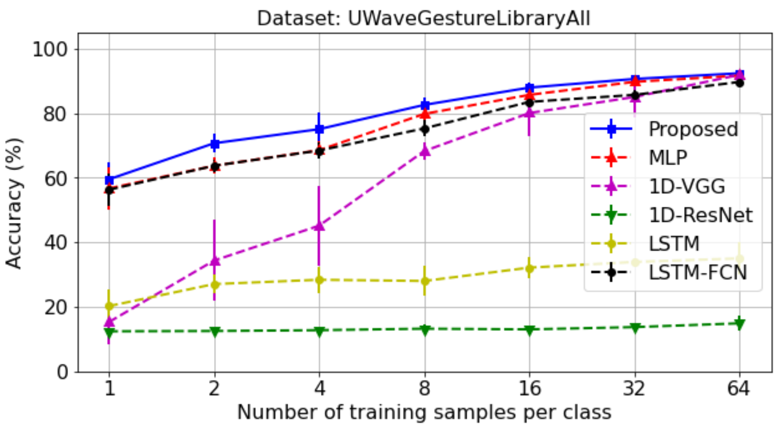

Solution is given by: $\underset{c}{\arg\min}\ d^2\left(\widehat{s}, \widetilde{\widehat{\mathbb{V}}}_m^{(c)}\right)$

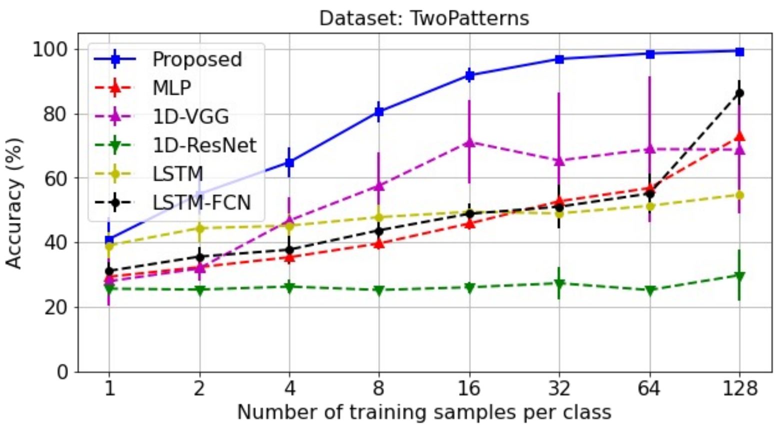

(a) GesturePebbleZ2
Class 1
Class 2
(b) InsectEPGregularTrain
Class 1
Class 2
(c) Trace
Class 1
Class 2
(d) PLAID
Class 1
Class 2
(e) UWaveGestureLibraryAll
Class 1
Class 2
(f) Wafer
Class 1
Class 2
(g) StarLightCurves
Class 1
Class 2
(h) TwoPatterns
Class 1
Class 2
(i) ECG5000
Class 1
Class 2
(j) MITBIH-AAMI
Class 1
Class 2

# Results and comparisons with deep learning

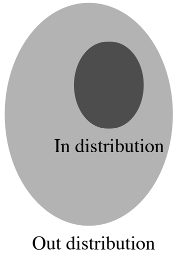
[Rubaiyat et al, ArXiv 2022]

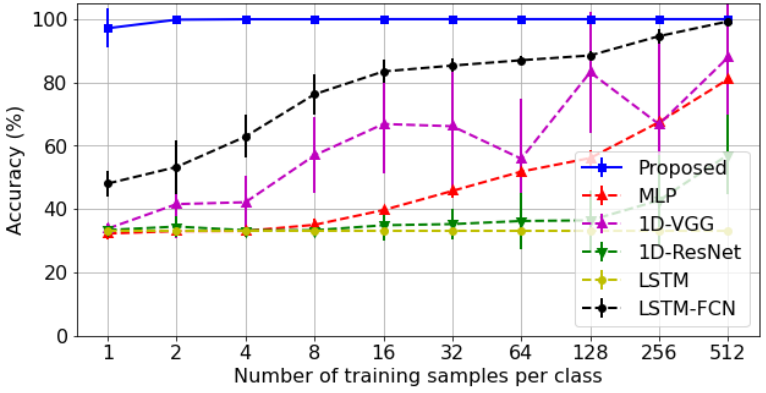
| Dataset | MLP | 1D-VGG | 1D-ResNet | LSTM | LSTM-FCN | SCDT-NS | **Proposed** |
|---|---|---|---|---|---|---|---|
| GesturePebbleZ2 | 83.92 | 48.99 | 63.23 | 60.19 | 85.82 | 81.64 | **86.07** |
| InsectEPGRegularTrain | 80.92 | 82.07 | 50.58 | 57.05 | 59.95 | 73.43 | **90.82** |
| Trace | 64.0 | 35.8 | 25.9 | 28.2 | 58.1 | 85.0 | **88.0** |
| PLAID | 39.65 | 19.35 | 29.81 | 26.35 | 39.57 | 58.29 | **70.02** |
| UWaveGestureLibraryAll | 94.74 | **96.42** | 42.86 | 30.72 | 94.16 | 90.4 | 94.92 |
| Wafer | 68.69 | 94.03 | 53.66 | 51.36 | 86.70 | 92.75 | **95.53** |
| StarLightCurves | 75.85 | 70.0 | 56.53 | 60.54 | 74.38 | 77.4 | **84.2** |
| TwoPatterns | 88.12 | 95.15 | 87.42 | 80.62 | 98.47 | 95.15 | **99.92** |
| ECG5000 | 98.9 | **98.92** | **98.92** | 98.36 | 98.76 | 93.4 | 97.4 |
| MITBIH-AAMI | 65.54 | 82.94 | 59.13 | 55.34 | 79.46 | 62.9 | **84.05** |
| Win | 0 | 2 | 1 | 0 | 0 | 0 | **8** |
| AVG arithmetic ranking | 3.5 | 3.5 | 5.6 | 6.3 | 3.6 | 3.8 | **1.6** |
| AVG geometric ranking | 3.42 | 2.8 | 5.07 | 6.27 | 3.46 | 3.49 | **1.28** |
| MPCE | 0.072 | 0.065 | 0.125 | 0.127 | 0.066 | 0.059 | **0.030** |




Out of distribution testing



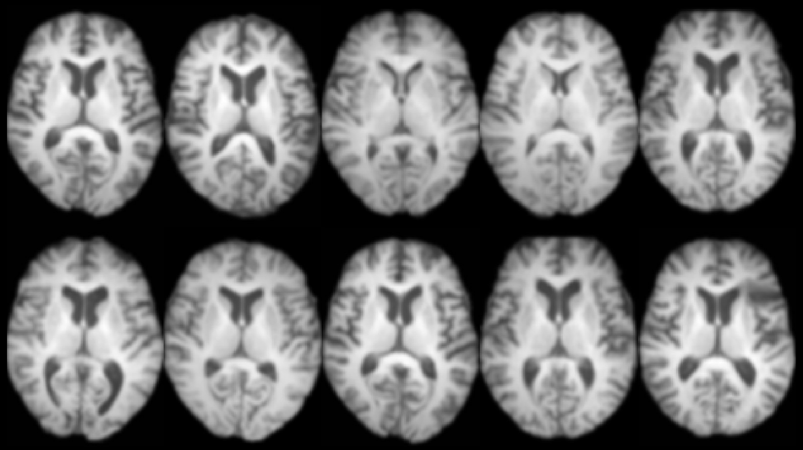
# Transport-based Morphometry (TBM)

# Biomedical image data science

Are fitness and gray matter morphology related?

Class 1:
persons with low fitness index

Class 2:
persons with high fitness index

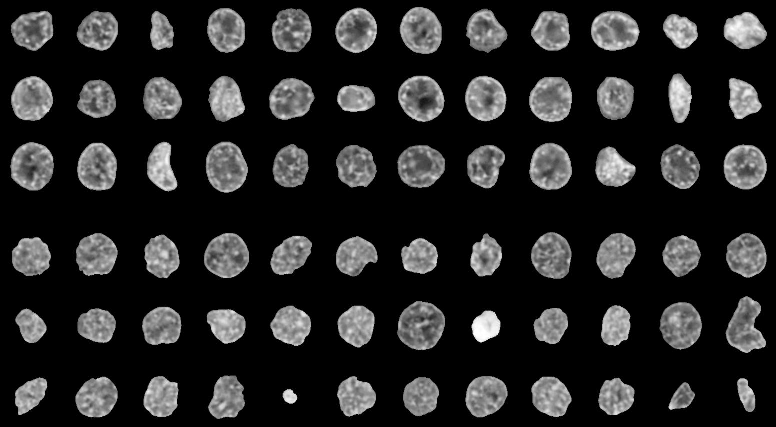


magnetic resonance imaging (MRI)

Is nuclear morphology related to cancer?

Class 1:
nuclei from benign tissues

Class 2:
nuclei from malignant tissues



digital pathology (microscopy)

## Hypothesis testing

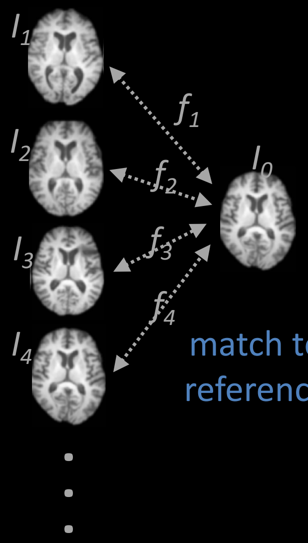
Are there differences?

## Understanding

If so, can we model them?

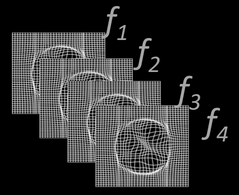
# Transport-based morphometry (TBM)

## 1) Transform




match to reference

transport maps




## 2) regression

modeling variations

- PCA, least squares, …

modeling differences

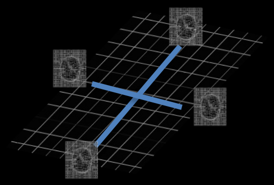
- LDA, SVM, …

modeling relationships

- CCA, …

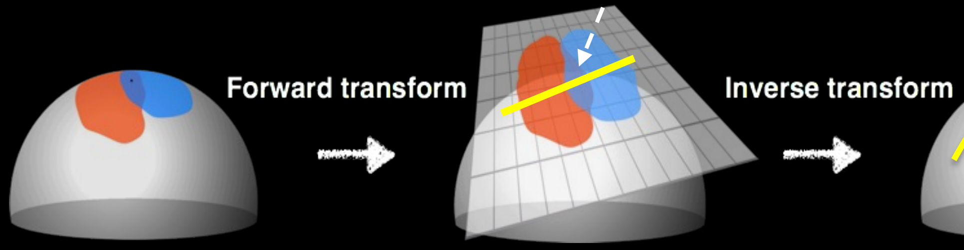
## 3) Inverse transform

PCA, LDA, CCA, …



visualization

statistical regression (PCA, LDA, CCA, …)



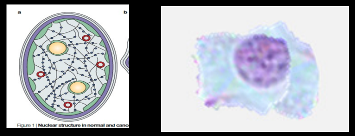

signal space

transform space

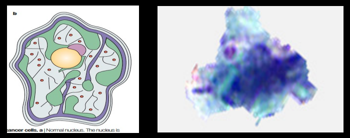
model in image space

# Example: nuclear structure in cancer

Aberrations in genetic code and/or increase in transcriptional activity → Unfolding/repackaging of chromatin → change in nuclear configuration

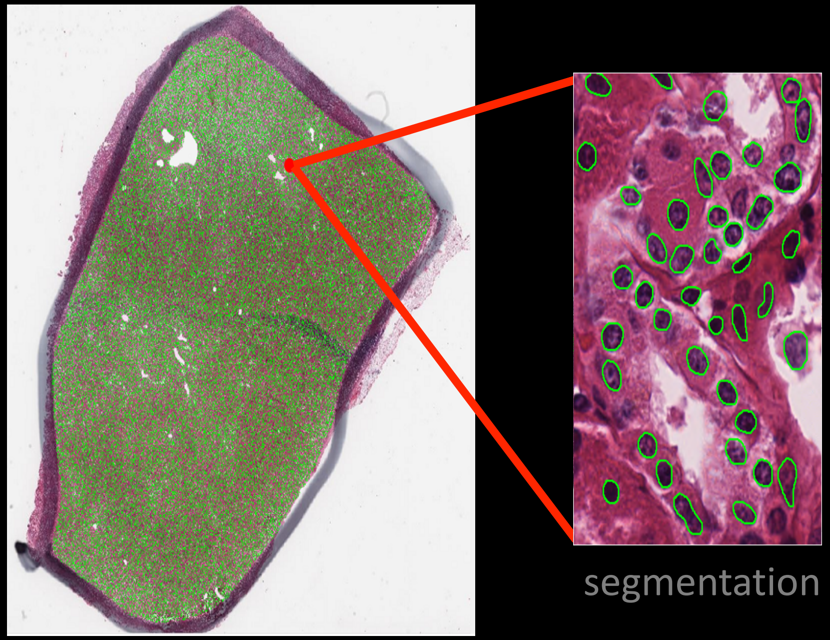


normal



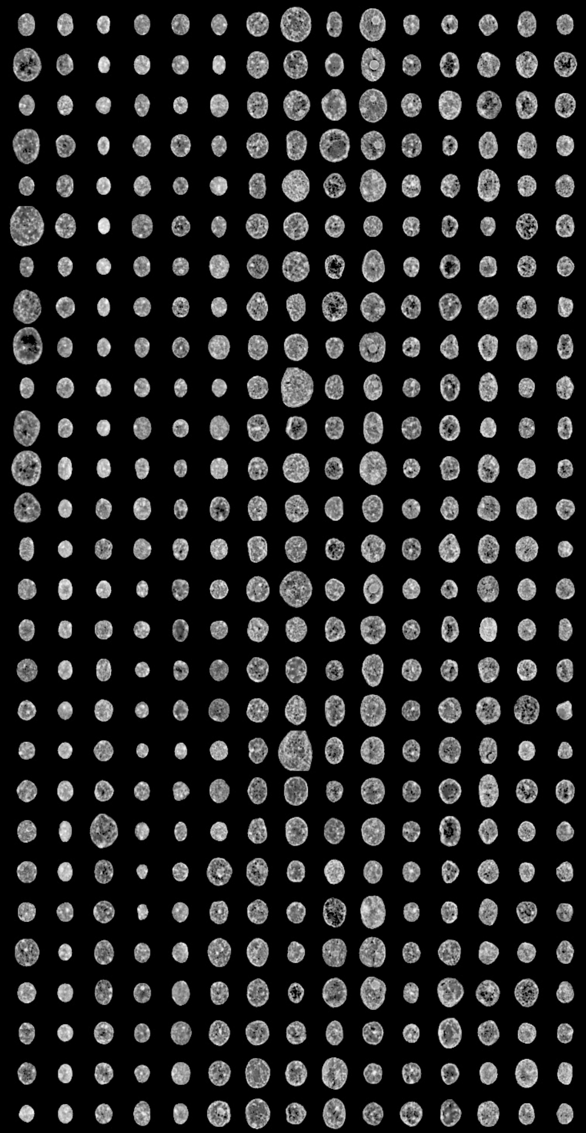
cancer

Pathology, whole slide imaging



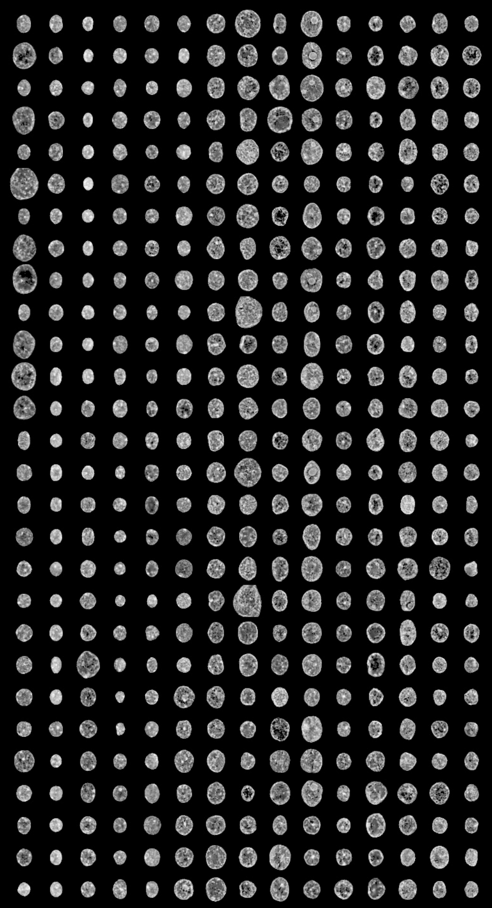
segmentation

segmented nuclei

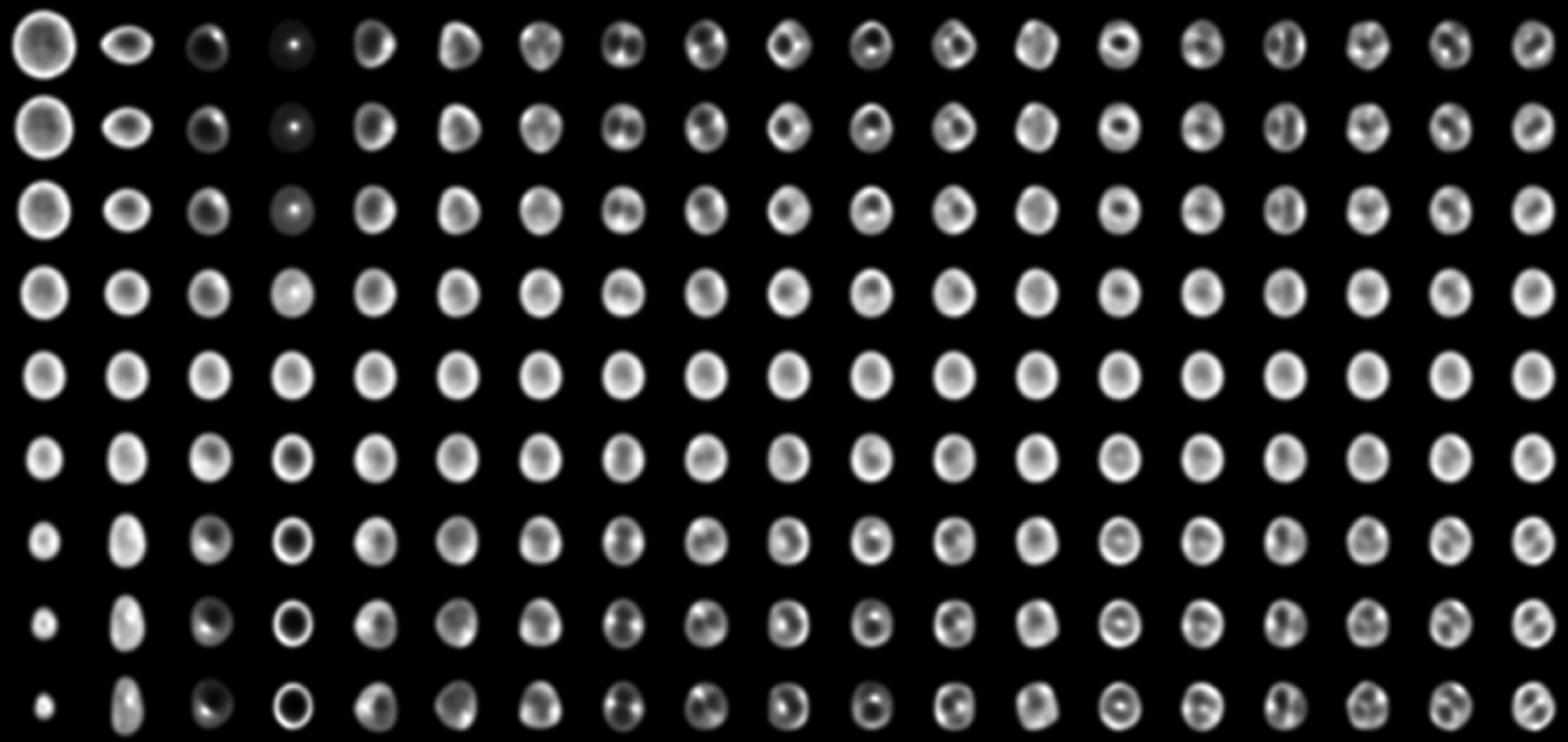


# Exploratory analysis: PCA on transport maps

database



## Modeling chromatin variation in liver cancer (shape + texture)



learned “features”

$$D_{f_\alpha^{-1}}(x)\, s_0(f_\alpha^{-1}(x))$$

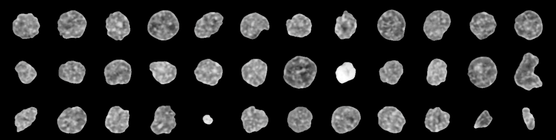
$$f_\alpha(x) = \underbrace{f(x)}_{\text{mean}} + \alpha w(x)$$

PCA eigenvector

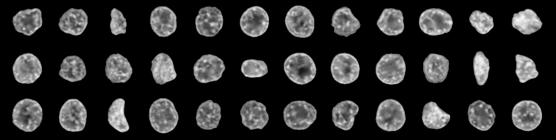
# TBM: modeling discriminant information

[Basu et al, PNAS 2014]

Class 1: benign cells

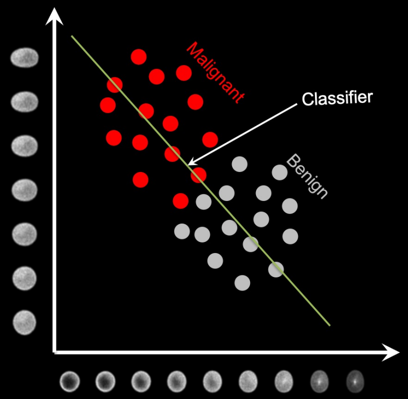


Class 2: malignant cells



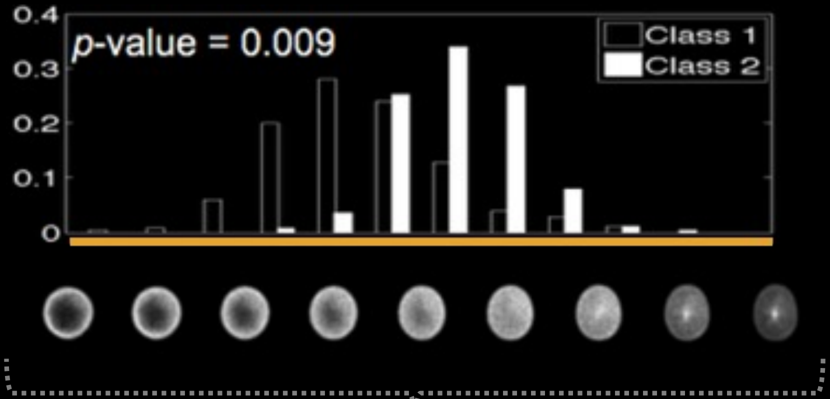
forward

**LDA in transport space**




inverse

Fetal-type hepatoblastoma




## Inverse of classification function

$$D_{f_\alpha^{-1}}(x)\, s_0(f_\alpha^{-1}(x))$$

$$f_\alpha(x) = f(x) + \alpha w(x)$$

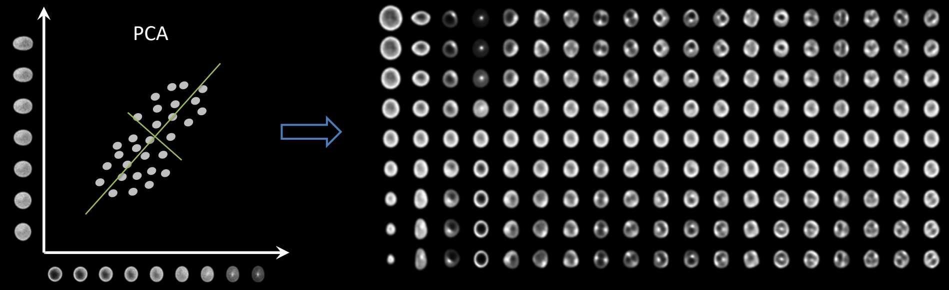
mean

classifier

# Summary: transport-based morphometry

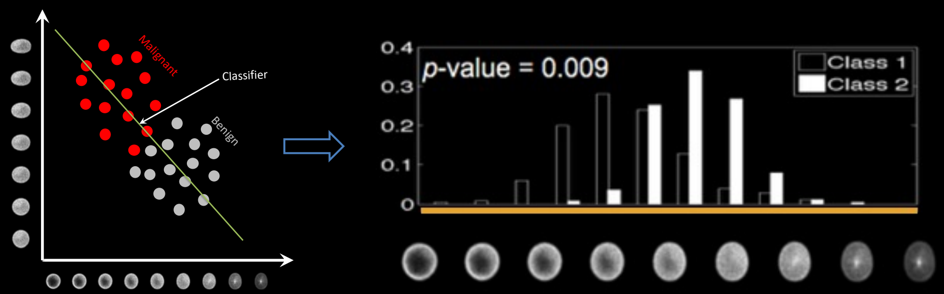
## 1) Exploratory analysis

- Transport PCA, subspace learning, …
- Models texture & shape simultaneously
- Visualizes variations within dataset



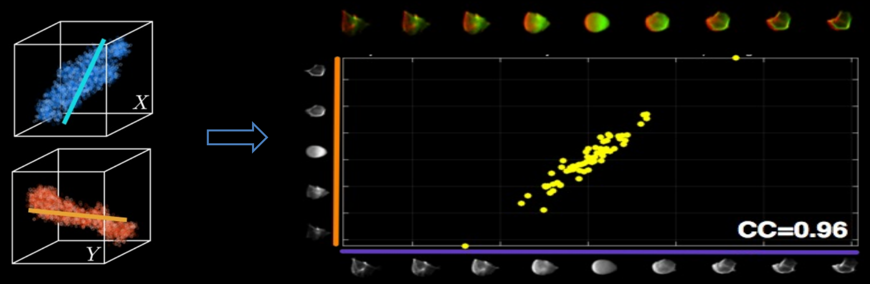

## 2) Discriminant models

- SVM, LDA, pLDA, …
- Visualizes most discriminative information




## 3) Functional relationships

- CCA
- Study & visualize relationship between different measurements



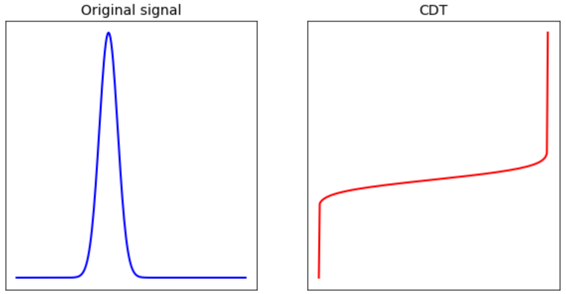
# PyTransKit

Python transport based signal processing toolkit

https://github.com/rohdelab/PyTransKit

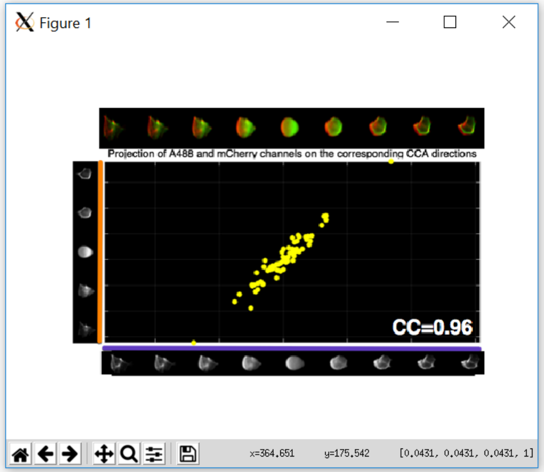
## Low level functions

```
x = np.arange(N)
mu = 100
sigma = 10
I1 = 1 / (simga*np.sqrt(2*np.pi))*np.exp(-((x-mu)**2)/(2*sigma**2))
I1 = abs(I1) + epsilon
I1 = I1/I1.sum()


# compute the forward transform
I1_hat, f1_hat, xtile = cdt.forward(x0, I0, x, I1)
```

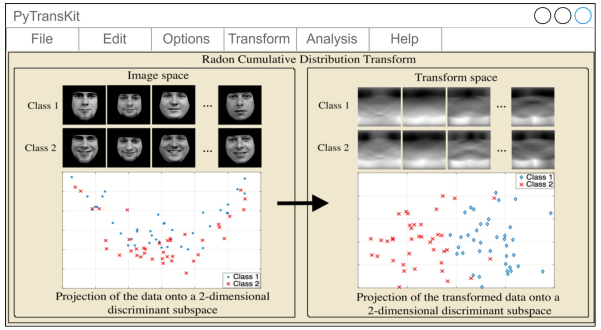



## Command line tools

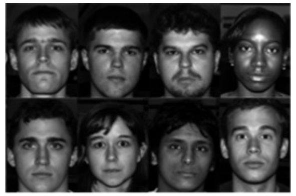
```
# load the neutrophils dataset
x, y = load_data('data/neutrophils')

# R-CDT transform with default parameters
x_t = radoncdt.transform(x)

# CCA analysis, using 2 components
x_t_comps = cca.fit_transform(x_t, y, n_components=2)

# Show results
plt.plot(x_t_comps, y)
```



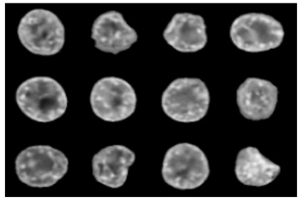

## TBM toolbox



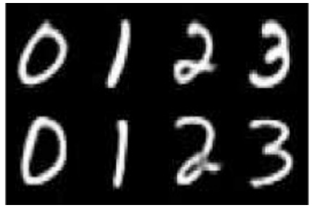
faces



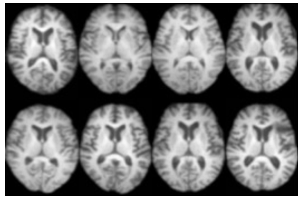
cells



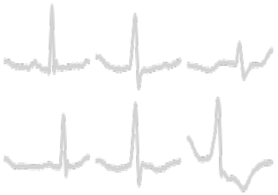
digits



brains



ECG



estimation, detection,
inverse problems, classification

| signal space | | transform space | | signal space |
|---|---|---|---|---|
| problem hard to solve | $\Rightarrow$ | problem easy to solve | $\Rightarrow$ | answer in signal space |

**signal space**

$$\mathbb{S}^{(c)} = \bigcup_{m=1}^{M_c} \mathbb{S}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}},$$

$$\mathbb{S}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}} = \left\{ s_{j,m}^{(c)} \middle| s_{j,m}^{(c)} = g_j' \varphi_m^{(c)} \circ g_j,\, g_j' > 0,\, g_j \in \mathcal{G}_m^{(c)} \right\}$$

**transform space**

$$\widehat{\mathbb{S}}^{(c)} = \bigcup_{m=1}^{M_c} \widehat{\mathbb{S}}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}},$$

$$\widehat{\mathbb{S}}_{\varphi_m^{(c)},\mathcal{G}_m^{(c)}} = \left\{ \widehat{s}_{j,m}^{(c)} \middle| \widehat{s}_{j,m}^{(c)} = g_j^{-1} \circ \widehat{\varphi}_m^{(c)},\, g_j \in \mathcal{G}_m^{(c)} \right\}$$